Cambridge Assessment
Admissions Testing

TEST OF MATHEMATICS
FOR UNIVERSITY ADMISSION

D513/11

2024 PAPER 2

Morning

Practice paper

Time: 75 minutes

Additional Materials: Answer sheet

INSTRUCTIONS TO CANDIDATES

Please read these instructions carefully, but do not open the question paper until you are told that you may do so.

A separate answer sheet is provided for this paper. Please check you have one. You also require a soft pencil and an eraser.

There are 20 questions on this paper. For each question, choose the one answer you consider correct and record your choice on the separate answer sheet. If you make a mistake, erase thoroughly and try again.

There are no penalties for incorrect responses, only points for correct answers, so you should attempt all 20 questions. Each question is worth one mark.

Any rough work should be done on this question paper. No extra paper is allowed.

Question 1

If $a+b, b+c, a-c$ are all even

What can be inferred about the parity of a, b, c?

A. a is even, b and c are odd

B. a and b are even, c is odd

C. a is odd, b is even, c is odd

D. Only one of a, b, c is even

E. a, b, c are all odd or all even

Question 2

An arithmetic sequence (S_n) is defined by its first term a_1, number of terms n, and common difference d. Consider the following statement: "If a_1 is odd, then all terms S_n are odd." Identify a counterexample for this statement from the options below.

A. $a_1 = 2, d = 2, n = 20$

B. $a_1 = 1, d = 3, n = 10$

C. $a_1 = 3, d = 6, n = 30$

D. $a_1 = 5, d = 4, n = 15$

E. $a_1 = 4, d = 5, n = 8$

Question 3

If the statement P: "For any n such that $n \not\equiv 1$ (mod6), n is not prime" is false, then which of the following is true?

A. There exist a $n \not\equiv 1 \pmod 6$ and n is prime.

B. There exist a $n \equiv 1 \pmod 6$ and n is prime.

C. For any prime $n, n \equiv 1$ (mod6)

D. For any prime $n, n \not\equiv 1$ (mod6)

Question 4

Find the number of positive integer solutions for the equation n which satisfy $((n-1)!)! = (n-1)!$

A. 0

B. 1

C. 2

D. 3

E. 4

F. 5

G. 6

Question 5

Laud Polygon: For any interior angle A, there exists an angle B such that $B = 2A$ or $B = \frac{A}{2}$.

A polygon is quiet if it is not a laud polygon.

Which of the following is/are quiet?

I: 30°, 60°, 90°

II: 40°, 80°, 80°, 160°

III: 60°, 120°, 120°, 120°, 120°

A. Only I

B. Only II

C. Only III

D. I and II

E. I and III

F. II and III

G. All of them

Question 6

For positive integers a, b and c, define three least common multiples: $L = \mathrm{lcm}(a, b)$, $M = \mathrm{lcm}(b, c)$, and $N = \mathrm{lcm}(a, c)$. Which of these is not possible?

A. $L < M < N$

B. $L < M = N$

C. $L = M < N$

D. $L = M = N$

E. All of them are possible

Question 7

Given that $0 \leq f(x) \leq 1$ for $0 < x < 4$, and that:

$$\int_0^2 f(x)\,dx = 1 \quad \text{and} \quad \int_1^3 f(x)\,dx = 1.5$$

Which of the following are possible values for $\int_0^4 f(x)\,dx$?

I: 2.4

II: 2.8

III: 3.2

A. Only I

B. Only II

C. Only III

D. I and II

E. I and III

F. II and III

G. All of them

Question 8

Let P be the statement "$f(x)$ is divisible by $(x+2)(x-3)$" and Q be the statement "$f(x)$ is divisible by $4x^2-4x-24$".

Determine what type of condition P is for Q.

- A. a sufficient condition for Q
- B. a sufficient but not necessary condition for Q
- C. a necessary and sufficient condition for Q
- D. neither a sufficient nor a necessary condition for Q
- E. Q is a necessary condition for P
- F. P is a sufficient and necessary condition for Q

Question 9

Which of the following is a necessary but insufficient condition for $sinx > \frac{1}{2}$?

- A. $cosx = 0$
- B. $\frac{1}{2}sinx < \frac{1}{4}$
- C. $tan\frac{x}{2} = 1$
- D. $cosx > \frac{\sqrt{3}}{2}$
- E. $sin^2x > \frac{9}{4}$

Question 10

For a cubic function $ax^3 + bx^2 + cx + d$, there are two local extrema, one at $x > 0$ and the other at $x < 0$. What is the necessary and sufficient condition for this to occur?

- A. $b > 3a$
- B. $b < 3a$
- C. $ac > 0$
- D. $ac < 0$
- E. $b^2 > 3ac$
- F. $b^2 > 4ac$

Question 11

Given the function $f(x) = x^3 + px^2 + q$, it has a local maximum at $x = 0$ and 3 distinct roots. Find the conditions for p and q.

A. $\frac{8}{27}p^3 < q < 0$

B. $0 < q < -\frac{8}{27}p^3$

C. $\frac{4}{27}p^3 < q < 0$

D. $0 < q < -\frac{4}{27}p^3$

Question 12

Given a geometric sequence (S_n) with the parity of first term a_1, the parity of the number of terms n, and the parity (odd or even nature) of the common ratio r, is it possible to determine the parity of the terms in the sequence (S_n)?

A. Possible

B. Impossible, need value of a_1

C. Impossible, need value of n

D. Impossible, need value of r

E. All of them are incorrect

Question 13

Given that $f'(x) < 0$ for $0 \leq x \leq 1$, which of the following statements must be true?

I. $\int_0^{\frac{1}{2}} f(x)\,dx > \int_{\frac{1}{2}}^{1} f(x)\,dx$

II. $\int_0^{\frac{1}{2}} f(x)\,dx < \int_0^{\frac{1}{2}} f(x^2)\,dx$

III. $f(0) < f(1)$

A. Only I

B. Only II

C. Only III

D. I and II

E. I and III

F. II and III

G. All of them

Question 14

Given that $f(x)=ax^2+bx+c$, which of the following is a sufficient condition for $f(|x|)=k$ to have 4 different real number solutions?

A. $b^2-4ac<0$

B. $b=0$

C. $ab<0$

D. $a>0$

E. $b^2>4ac$

Question 15

A car starts from depot and moves repeatedly between 4 stations in the following pattern: 1, 2, 3, 4, 3, 2, 1, 2, 3, ...It follows the below statements:

Statement 1：If the car has stopped at Station 1 a total of 10 times, it must have stopped at Station 2 exactly 20 times.

Statement 2：If the car has stopped at Station 3 a total of 10 times, it must have stopped at Station 4 exactly 5 times.

Statement 3：If the car has stopped at Station 2 a total of 10 times, it must have stopped at Station 3 exactly 10 times.

Determine which of the following statements are correct.

A. Only statement 1

B. Only statement 2

C. Only statement 3

D. Statement 2 and 3

E. Statement 1 and 2

F. Statement 1 and 3

G. All statements

Question 16

Given the sequence $u_{n+1} = 3u_n - 6$ with $u_n \in Z$, which of the following statements is true?

1. If $6|u_6$, then $3|u_3$.

2. If $10|u_{10}$, then $30|u_{30}$.

- A. Only statement 1
- B. Only statement 2
- C. Both statements
- D. Neither statement

Question 17

Given the function $f(x) = \frac{a}{x} + bx^2$, determine which of the following conditions are true if $f(x)$ has a local minimum and a positive root.

- A. $a > 0, b > 0$
- B. $a > 0, b < 0$
- C. $a < 0, b > 0$
- D. $a < 0, b < 0$

Question 18

The expansion of the expression $(x^p + \frac{1}{x^q})^n$ has a constant term.

Which of the following is/are necessary and sufficient conditions for this statement for some integer k?

I: $kp + kq = n$

II: $kp + kq = nq$

III: $kp + kq = np$

- A. Only I
- B. Only II
- C. Only III
- D. I and II
- E. I and III
- F. II and III
- G. All of them

Question 19

Define the left branch of a quadratic function as the portion to the left of its vertex, and similarly, define the right branch. Let $f(x)$ and $g(x)$ be two quadratic functions.

Statement P: If $g(x)$ is a translation of $f(x)$, then the left branch of $g(x)$ and the left branch of $f(x)$ intersect at one point, and the right branch of $g(x)$ and the right branch of $f(x)$ intersect at one point.

Which of the following is correct?

A. Statement P is correct

B. The converse of Statement P is correct

C. The contrapositive of Statement P is correct

D. None of them are correct

Question 20

Consider the following attempt to solve an equation. The steps have been numbered for reference.

Equation: $3\cos x = \sqrt{x}$, where x is in radians.

Solving Steps:

(1) Start with the equation: $3\cos x = \sqrt{x}$.

(2) Square both sides of the equation: $9\cos^2 x = x$.

(3) Use the identity $\cos^2 x = \frac{1}{2}(1+\cos 2x)$ to rewrite the equation as $9 \cdot \frac{1}{2}(1+\cos 2x) = x$.

(4) Graph the functions $y = \frac{1}{2}(1+\cos 2x)$ and $y = \frac{x}{9}$.

(5) From the graph, observe that at $x = \pi, y = \pi/9 \approx 0.35$ is less than $y = \frac{1}{2}(1+\cos 2\pi) = 1$, and at $x = 2\pi, y = 2\pi/9 \approx 0.70$ is less than 1. The graphs intersect at five points.

(6) Conclude that the equation has five solutions.

Which one of the following statements is true?

A. All five solutions are solutions of the original equation.

B. None of the five solutions is a solution of the original equation.

C. Only three solutions are valid, and the incorrect solutions arise as a result of step (1).

D. Only three solutions are valid, and the incorrect solutions arise as a result of step (2).

E. Only three solutions are valid, and the incorrect solutions arise as a result of step (3).

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Test of Mathematics for University Admission (TMUA) 2024

Paper 2 - answer key

Question	Answer
1	E
2	B
3	A
4	D
5	F
6	C
7	D
8	B
9	C
10	D
11	D
12	A
13	D
14	C
15	D
16	C
17	C
18	F
19	D
20	D