Beyond Horizon TMUA Mock Paper

Set 3 Paper 2

75 Minutes

20 Questions

One answer per question

1) Which of these numbers is the largest?

(A) $\frac{2+\sqrt{5}}{5\sqrt{2}}$

(B) $\frac{\sqrt{9!}}{6!}$

(C) $\frac{\log_3 26}{\log_2 33}$

(D) $\frac{\sqrt{7}}{5}$

(E) $\frac{\pi}{5}$

2) The graph of the function $y = 2^{x^2-4x+3}$ can be obtained from the graph of $y = 2^{x^2}$ by

(A) a stretch parallel to the y-axis followed by a translation parallel to the y-axis

(B) a translation parallel to the x-axis followed by a stretch parallel to the y-axis

(C) a translation parallel to the x-axis followed by a stretch parallel to the x-axis

(D) a translation parallel to the x-axis followed by reflection in the y-axis

(E) reflection in the y-axis followed by translation parallel to the y-axis

3) Find the highest power of x in

$$\left[\left((2x^5+6)^4+(4x^6+3)^3\right)^7+\left((2x^4+3x^5)^5-(x^7-6)^4\right)^5\right]^{10}$$

(A) 1200

(B) 1300

(C) 1400

(D) 1500

(E) 1600

4) Three positive numbers a, b, c satisfy $\log_b a = 2, \log_b (c-3) = 3$, $\log_a (c+5) = 2$. This information

(A) specifies a uniquely

(B) is satisfied by two values of a

(C) is satisfied by infinitely many values of a

(D) is contradictory

5) Which of these people A-E must be telling the truth if they each speak as follows?

(A) I'm not a liar

(B) I'm the only liar

(C) If E is a liar, so is D

(D) If A is a liar, so am I

(E) None of us are liars

6) (*) is the statement that the functions

$$f(x) = b^{ax}$$

$$g(x) = b^{x^a}$$

$$h(x) = b^{a^x}$$

are all defined and increasing for $x > 0$. Find the range of a and b for which (*) is true.

(A) $a > 0$ and $b > 0$

(B) $a < 0$ and $b > 1$

(C) $a > 1$ and $b > 0$

(D) $a > 1$ and $b > 1$

(E) $a > 0$ and $0 < b < 1$

(F) $a = 1$ and $b > 0$

7) Each of the positive real numbers a, b, c, d, e is decreased by 20%. Find the resulting percentage change in the value of the following expression:

$$\frac{a-b}{cd} - \frac{b^2+c^2}{d^3+ae^2}$$

(A) No change

(B) Decrease of 20%

(C) Increase of 20%

(D) Decrease of 25%

(E) Increase of 25%

8) Let f be a linear function with the properties that $f(1) \leq f(2), f(3) \geq f(4)$, and $f(5) = 5$. Which of the following is true?

(A) $f(0) < 0$

(B) $f(0) = 0$

(C) $f(1) < f(0) < f(-1)$

(D) $f(0) = 5$

(E) $f(0) > 5$

9) When n standard 6-sided dice are rolled, the probability of obtaining a sum of 1994 is greater than zero and is the same as the probability of obtaining a sum of S. The smallest possible value of S is

(A) 333

(B) 335

(C) 337

(D) 339

(E) 341

10) A hockey team consists of 1 goalkeeper, 4 defenders, 4 midfielders and 2 forwards. There are 4 substitutes: 1 goalkeeper, 1 defender, 1 midfielder and 1 forward. A substitute may only replace a player of the same category (e.g., midfielder for midfielder). Given that a maximum of 3 substitutes may be used and that there are still 11 players on the pitch at the end, how many different teams could finish the game?

(A) 110

(B) 118

(C) 121

(D) 125

(E) 132

11) Consider the unit circle $x^2 + y^2 = 1$. The locus of a point P such that the tangents PA, PB at the points A, B respectively of the circle are so that $\angle AOB = 60°$, where O is the origin, is

(A) a circle of radius $\frac{2}{\sqrt{3}}$ with centre O

(B) a circle of radius $\sqrt{3}$ with centre O

(C) a circle of radius 2 with centre O

(D) a pair of straight lines

12) Let l, m, n be any three positive integers such that $l^2 + m^2 = n^2$. Then,

(A) 3 always divides mn

(B) 3 always divides lm

(C) 3 always divides ln

(D) 3 does not divide lmn

13) Suppose $n \geq 2$ is a fixed positive integer and $f(x) = x^n|x|, x \in R$. Then

(A) f is differentiable everywhere only when n is even

(B) f is differentiable everywhere except at 0 if n is odd

(C) f is differentiable everywhere

(D) none of the above is true

14) There are 100 people in a queue waiting to enter a hall. The hall has exactly 100 seats numbered from 1 to 100. The first person in the queue enters the hall, chooses any seat and sits there. The n-th person in the queue, where n can be 2, …, 100, enters the hall after the $(n-1)$-th person is seated. He sits in seat number n if he finds it vacant; otherwise he takes any unoccupied seat. Find the total number of ways in which 100 seats can be filled up, provided the 100-th person occupies seat number 100.

(A) 100

(B) 100^{100}

(C) 2^{98}

(D) 2^{99}

(E) 2^{100}

15) Find an expression for $\int_1^n (-1)^{\lfloor x \rfloor} \lfloor x \rfloor^{-1} \, dx$ where $n \in N$.

(A) $\sum_{k=1}^{n} (-1)^k \frac{1}{k}$

(B) $\sum_{k=1}^{n-1} (-1)^k \frac{1}{k}$

(C) $\sum_{k=1}^{n-1} (-1)^k \frac{1}{k^2}$

(D) $\sum_{k=1}^{n} (-1)^k \frac{1}{k^2}$

16) The mean, median, unique mode, and range of a collection of eight integers are all equal to 8. The largest integer that can be an element of this collection is

(A) 11

(B) 12

(C) 13

(D) 14

(E) 15

17) For all x such that $1 \leq x \leq 3$, the inequality $(x - 3a)(x - a - 3) < 0$ holds for

(A) no value of a

(B) all a satisfying $\frac{2}{3} < a < 1$

(C) all a satisfying $0 < a < \frac{1}{3}$

(D) all a satisfying $\frac{1}{3} < a < \frac{2}{3}$

(E) all a satisfying $0 < a < \frac{2}{3}$

(F) all a satisfying $0 < a < 1$

18) A set of tiles numbered 1 through 100 is modified repeatedly by the following operation: remove all tiles numbered with a perfect square, and renumber the remaining tiles consecutively starting with 1. How many times must the operation be performed to reduce the number of tiles in the set to one?

(A) 10

(B) 11

(C) 18

(D) 19

(E) 20

19) If a and b are positive numbers and c and d are real numbers, positive or negative, then $a^c \leq b^d$

(A) if $a \leq b$ and $c \leq d$

(B) if either $a \leq b$ or $c \leq d$

(C) if $a \geq 1$, $b \geq 1$, $d \geq c$

(D) if $a \leq 2$, $b \leq 2$, $d \geq 2$

(E) if $a \leq 0$, $b \leq 1$, $d \leq c$

(F) is not implied by any of the above conditions

20) Consider the triangular array of numbers with 0, 1, 2, 3, ... along the sides and interior numbers obtained by adding the two adjacent numbers in the previous row. Rows 1 through 6 are shown:

$$
\begin{array}{ccccccccccc}
 & & & & & 0 & & & & & \\
 & & & & 1 & & 1 & & & & \\
 & & & 2 & & 2 & & 2 & & & \\
 & & 3 & & 4 & & 4 & & 3 & & \\
 & 4 & & 7 & & 8 & & 7 & & 4 & \\
5 & & 11 & & 15 & & 15 & & 11 & & 5
\end{array}
$$

Let $f(n)$ denote the sum of the numbers in row n. What is the remainder when $f(100)$ is divided by 100?

(A) 12

(B) 30

(C) 50

(D) 62

(E) 74