Beyond Horizon TMUA Mock Paper

Set 3 Paper 1

75 Minutes

20 Questions

One answer per question

1) Consider the inequality $\sqrt{x+3} - \sqrt{x-1} \geq 1$. Find the range of x that satisfies this inequality.

(A) $[-2, 3.25]$

(B) $[-1, 2.5]$

(C) $[1, 3]$

(D) $[2, 3.5]$

(E) $[-1, 2]$

(F) $[0, 3]$

(G) $[1, 3.25]$

2) Let $d_1, d_2, \ldots, d_k$ be all the factors of a positive integer n including 1 and n. Suppose $d_1 + d_2 + \cdots + d_k = 72$. Find the value of

$$\frac{1}{d_1} + \frac{1}{d_2} + \cdots + \frac{1}{d_k}$$

(A) $\frac{k^2}{72}$

(B) $\frac{72}{k}$

(C) $\frac{72}{n}$

(D) $\frac{72}{k^2}$

(E) $\frac{72}{n^2}$

(F) cannot be found from the given information

3) How many values of x satisfy the equation

$$3\cos^2 x + 4\sin x = 3$$

in the range $0 \leq x < 2\pi$?

(A) 0

(B) 1

(C) 2

(D) 3

(E) 4

(F) 5

4) Which power of x which has the greatest coefficient in the expansion of

$$\left(1+\frac{2}{3}x\right)^{12}$$

(A) x^3

(B) x^4

(C) x^5

(D) x^6

(E) x^7

(F) x^8

5) Which of the following numbers is the largest in value?

(A) $\tan\left(\frac{5\pi}{4}\right)$

(B) $\sin^2\left(\frac{5\pi}{4}\right)$

(C) $\log_{10}\left(\frac{5\pi}{4}\right)$

(D) $\log_2\left(\frac{5\pi}{4}\right)$

(E) $\log_3\left(\frac{5\pi}{4}\right)$

(F) $\tan\left(\frac{3\pi}{4}\right)$

6) If $2\log(\alpha - 2\beta) = \log(\alpha) + \log(\beta)$, find the value of $\frac{\alpha}{\beta}$.

(A) 1

(B) 2

(C) 3

(D) 4

(E) 1 and 4

(F) 2 and 3

7) Three boys of class I, 4 boys of class II and 5 boys of class III sit in a row. The number of ways they can sit, so that boys of the same class sit together is

(A) $3!4!5!$

(B) $\frac{(12)!}{3!4!5!}$

(C) $(3!)^2 4!5!$

(D) $3 \times 4!5!$

(E) $(4!)^2 5!6!$

(F) $3!(4!)^2 5!$

8) The area enclosed between the line $y = kx$ and the curve $y = x^3$ is 8. What is the value of k?

(A) 2

(B) 4

(C) $\sqrt{2}$

(D) $\sqrt{5}$

(E) $4\sqrt{2}$

(F) $2\sqrt{5}$

9) The coefficient of x^4 in the expansion of $(1 + x - 2x^2)^7$ is

(A) -81

(B) -91

(C) 81

(D) 91

(E) 101

(F) -101

10) The curve $y = 2x^3 - 9x + 5$ has turning points at $x = \alpha$ and $x = \beta$, where $\beta > \alpha$. Find

$$\int_{\alpha}^{\beta} 2x^3 - 9x + 5\,dx$$

(A) $-16\sqrt{2}$

(B) -20

(C) $-20 + 12\sqrt{2}$

(D) 0

(E) $5\sqrt{2}$

(F) $5\sqrt{6}$

(G) $20\sqrt{2}$

11) A rectangle is drawn in the region enclosed by the curves p and q, where

$$p(x) = 10 - 3x^2$$

$$q(x) = x^2 - 3$$

such that the sides of the rectangle are parallel to the x- and y-axes.

What is the maximum possible area of the rectangle?

(A) $\frac{26}{9}$

(B) $\frac{26\sqrt{39}}{9}$

(C) $\frac{5\sqrt{39}}{3}$

(D) $\frac{10\sqrt{6}}{3}$

(E) $5\sqrt{39}$

(F) $10\sqrt{6}$

12) Use the trapezium rule with 3 strips to estimate

$$\int_1^3 3 \log_{10} x \, dx$$

(A) $\log_{10} \frac{3\sqrt{2}}{4}$

(B) $\log_{10} \frac{1225}{9}$

(C) $\log_{10} \frac{27}{8}$

(D) $\log_{10} 4$

(E) $\log_{10} \frac{1225}{27}$

(F) $\log_{10} \frac{3\sqrt{17}}{4}$

13) A club with x members is organized into four committees according to the following rules:

1. Each member belongs to exactly two committees.
2. Each pair of committees has exactly one member in common.

Then

(A) $x = 4$;

(B) $x = 6$;

(C) $x = 8$;

(D) x cannot be determined from the given information.

(E) $x = 10$;

(F) $x = 12$;

14) If α and β are roots of the equation $x^2+5x-5=0$, then $\left(\frac{1}{\alpha+1}\right)^3+\left(\frac{1}{\beta+1}\right)^3$ equals

(A) -322

(B) $\frac{4}{27}$

(C) $-\frac{4}{27}$

(D) $3+\sqrt{5}$

(E) $4+\sqrt{5}$

(F) $5+\sqrt{2}$

15) The functions f, g, and h are related by

$$f'(x) = g(x+2)$$

$$g'(x) = h(x-2)$$

It follows that $f''(3x)$ equals

(A) $h(3x+2)$

(B) $3h'(3x)$

(C) $h(3x)$

(D) $9h(3x)$

(E) $6h'(3x)$

(F) $h(3x-2)$

16) The letters of the word "MOTHER" are permuted, and all the permutations so formed are arranged in alphabetical order as in a dictionary. Then the number of permutations which come before the word "MOTHER" is

(A) 503

(B) 93

(C) $\frac{6!}{2} - 1$

(D) 308

(E) 245

(F) 182

17) The number of terms in the expansion of $(x+y+z+w)^{10}$ is

(A) $\binom{10}{4}$

(B) $\binom{13}{3}$

(C) $\binom{14}{4}$

(D) 11^4

(E) $\binom{12}{2}$

(F) $\binom{15}{3}$

18) Let $\{F_n\}$ be the sequence of numbers defined by $F_1 = 1 = F_2$; $F_{n+1} = F_n + F_{n-1}$ for $n \geq 2$. Let f_n be the remainder left when F_n is divided by 5. Then f_{2000} equals

(A) 0

(B) 1

(C) 2

(D) 3

(E) 4

19) What is the value of the expression

$$1 + \frac{1}{2}\binom{n}{1} + \frac{1}{3}\binom{n}{2} + \cdots + \frac{1}{n+1}\binom{n}{n}$$

(A) $\frac{2^{n+1}-1}{n+1}$

(B) $\frac{2(2^n-1)}{n+1}$

(C) $\frac{2^n-1}{n}$

(D) $\frac{2(2^{n+1}-1)}{n+1}$

(E) $\frac{2^{n+2}-1}{n+2}$

(F) $\frac{2(2^n-1)}{n}$

20) What is the value of this expression ($q \neq 1$)

$$(1+q)(1+q^2)(1+q^4)(1+q^8)(1+q^{16})(1+q^{32})(1+q^{64}),$$

(A) $\frac{1-q^{128}}{1-q}$

(B) $\frac{1-q^{64}}{1-q}$

(C) $\frac{1-q^{2^{1+2+\cdots+6}}}{1-q}$

(D) none of the foregoing expressions;

(E) $\frac{1-q^{2^{64}}}{1-q}$

(F) $\frac{1-q^{128}}{1+q}$