Beyond Horizon TMUA Mock Paper

Set 2 Paper 2

75 Minutes

20 Questions

One answer per question

1) $3x^4 - 6x^3 + kx^2 - 8x - 12$ is divisible by $x - 3$, so it is also divisible by

(A) $3x^2 - 4$

(B) $3x^2 + 4$

(C) $3x^2 + x$

(D) $3x^2 - x$

(E) $4x^2 + x$

(F) $4x^2 - x$

2) Consider the statement: $x(\alpha - x) < y(\alpha - y)$ for all x, y with $0 < x < y < 1$. The statement is true

(A) if and only if $\alpha \geq 2$

(B) if and only if $\alpha > 2$

(C) if and only if $\alpha \leq -1$

(D) if and only if $\alpha \geq -1$

(E) if and only if $\alpha \leq 2$

(F) for no values of α

3) A television station telecasts three types of programs X, Y, and Z. A survey gives the following data on television viewing. Among the people interviewed 60% watch program X, 50% watch program Y, 50% watch program Z, 30% watch programs X and Y, 20% watch programs Y and Z, 30% watch programs X and Z, while 10% do not watch any television program. The percentage of people watching all the three programs X, Y, and Z is

(A) 90

(B) 50

(C) 10

(D) 20

(D) 30

(D) 40

4) In a football league, a particular team played 60 games in a season. The team never lost three games consecutively and never won five games consecutively in that season. If N is the number of games the team won in that season, then N satisfies

(A) $24 \leq N \leq 50$

(B) $20 \leq N \leq 48$

(C) $12 \leq N \leq 40$

(D) $18 \leq N \leq 42$

(E) $20 \leq N \leq 42$

(F) $24 \leq N \leq 42$

5) Every integer of the form $(n^3 - n)(n - 2)$, for $n = 3, 4, \ldots$, is

(A) divisible by 6 but not always divisible by 12

(B) divisible by 12 but not always divisible by 24

(C) divisible by 24 but not always divisible by 48

(D) divisible by 9

(E) divisible by 48 but not always divisible by 96

(F) divisible by 5

6) From a group of seven persons, seven committees are formed. Any two committees have exactly one member in common. Each person is in exactly three committees. Then

(A) at least one committee must have more than three members

(B) each committee must have exactly three members

(C) each committee must have more than three members

(D) nothing can be said about the sizes of the committees

7) Given any five points in the square $I^2 = \{(x, y) : 0 \leq x \leq 1, 0 \leq y \leq 1\}$, only one of the following statements is true. Which one is it?

(A) The five points lie on a circle.

(B) At least one square can be formed using four of the five points.

(C) At least three of the five points are collinear.

(D) There are at least two points such that the distance between them does not exceed $\frac{1}{\sqrt{2}}$.

8) Let A_1, A_2, A_3 be three points on a straight line. Let B_1, B_2, B_3, B_4, B_5 be five points on a straight line parallel to the first one. Each of the three points on the first line is joined by a straight line to each of the five points on the second line. Further, no three or more of these joining lines meet at a point except possibly at the A's or the B's. Then the number of points of intersections of the joining lines lying between the two given straight lines is

(A) 30

(B) 25

(C) 35

(D) 20

9) Let $y = \log_a x$ and $a > 1$. Then only one of the following statements is false. Which one is it?

(A) If $x = 1$, then $y = 0$

(B) If $x < 1$, then $y < 0$

(C) If $x = \frac{1}{2}$, then $y = \frac{1}{2}$

(D) If $x = a$, then $y = 1$

10) If x is a real number and $y = \frac{1}{2}(e^x - e^{-x})$, then

(A) x can be either $\log(y + \sqrt{y^2 + 1})$ or $\log(y - \sqrt{y^2 + 1})$

(B) x can only be $\log(y + \sqrt{y^2 + 1})$

(C) x can be either $\log(y + \sqrt{y^2 - 1})$ or $\log(y - \sqrt{y^2 - 1})$

(D) x can only be $\log(y + \sqrt{y^2 - 1})$

11) If the roots of

$$\frac{1}{x+a}+\frac{1}{x+b}=\frac{1}{c}$$

are equal in magnitude but opposite in sign, then the product of the roots is

(A) $\frac{a^2+b^2}{2}$

(B) $\frac{-a^2+b^2}{4}$

(C) $\frac{a+b}{2}$

(D) $\frac{a^2+b^2}{2}$

12) How many positive real roots does this equation have?

$$x^4 - 2\sqrt{2}x^3 + 2x^2 - 4x = 0.$$

(A) 0

(B) 1

(C) 2

(D) 3

(E) 4

13) What is the digit in the unit position of the integer

$$1! + 2! + 3! + \cdots + 99!$$

(A) 3

(B) 0

(C) 1

(D) 7

(D) 5

(D) 9

14) Let $\{a_k\}$ be a sequence of integers such that $a_1 = 1$ and $a_{m+n} = a_m + a_n + mn$, for all positive integers m and n. Then a_{12} is

(A) 45

(B) 56

(C) 67

(D) 78

(E) 89

15) If $l^2 + m^2 + n^2 = 1$ and $p^2 + q^2 + r^2 = 1$, then the range of values $lp + mq + nr$ can take is

(A) $[2, \infty)$

(B) $[2, 1]$

(C) $(\infty, 1]$

(D) $[-1, 1]$

(E) $[1, \infty)$

(F) does not satisfy any of the above conditions

16) The function $f(x)$ is defined as follows:

$$f(z) = a_0 + a_1 x + a_n x^n, \quad a_i \in Z$$

Which of the following is true?

(A) There exists some odd n such that there exists some $a_0, a_1, \ldots, a_n$ such that $f(\sqrt{2} + \sqrt{3}) = 0$

(B) If $f(\sqrt{2} + \sqrt{3}) = 0$, then $f(\sqrt{2} - \sqrt{3}) = 0$

(C) For every even n, there exists some $a_0, a_1, \ldots, a_n$ such that $f(\sqrt{2}+\sqrt{3}) = 0$

(D) For every odd n, there exists some $a_0, a_1, \ldots, a_n$ such that $f(\sqrt{2}+\sqrt{5}) = 0$

17) Using 4 fours and just $+, -, *, /$ operations, in how many ways can the number 16 be generated?
E.g. 12 can be generated like this: $12 = 4 * (4 - 4/4)$
E.g. 8 can be generated like this: $8 = 4 + 4 - 4 + 4$
Note: the order of operations does matter, so $12 = 4 * (4 - 4/4)$ and $12 = (4 - 4/4) * 4$ are counted as 2 distinct generations.

(A) 4

(B) 5

(C) 8

(D) 10

(E) 13

(F) 19

(G) 25

18) Mr. Earl E. Bird leaves home every day at 8:00 AM to go to work. If he drives at an average speed of 40 miles per hour, he will be late by 3 minutes. If he drives at an average speed of 60 miles per hour, he will be early by 3 minutes. How many miles per hour does Mr. Bird need to drive to get to work exactly on time?

(A) 45

(B) 48

(C) 50

(D) 55

(E) 58

19) The set of all real numbers x for which

$$\log_{2004}(\log_{2003}(\log_{2002}(\log_{2001} x)))$$

is defined is $\{x \mid x > c\}$. What is the value of c?

(A) 0

(B) 2001^{2002}

(C) 2002^{2003}

(D) 2003^{2004}

(E) $2001^{2002^{2003}}$

20) A base-10 three-digit number n is selected at random. Which of the following is closest to the probability that the base-9 representation and the base-11 representation of n are both three-digit numerals?

(A) 0.3

(B) 0.4

(C) 0.5

(D) 0.6

(E) 0.7