Beyond Horizon TMUA Mock Paper

Set 2 Paper 1

75 Minutes

20 Questions

One answer per question

1) How many solutions does the equation $x \tan x = 2$ have in the interval $-3\pi \leq x \leq 3\pi$?

(A) 1

(B) 2

(C) 3

(D) 4

(E) 5

(F) 6

(G) 7

2) The least possible value of the gradient of the curve $y = (3x+b)(x-b)^2$ at the point where $x = 2$, as b varies, is:

(A) $-\dfrac{81}{4}$

(B) -64

(C) $-\dfrac{36}{4}$

(D) $\dfrac{5}{4}$

(E) $\dfrac{53}{16}$

3) It is given that

$$y = (1 + \cos x) \sin 3x \quad \text{for} \quad 0 < x < \pi$$

The complete set of values of x for which y is negative is:

(A) $0 < x < \frac{\pi}{6}$

(B) $0 < x < \frac{\pi}{3}$

(C) $\frac{\pi}{3} < x < \frac{\pi}{2}$

(D) $\frac{2\pi}{3} < x < \pi$

(E) $0 < x < \frac{\pi}{2}$

(F) $\frac{\pi}{6} < x < \frac{\pi}{2}$

4) The function $\frac{2-x}{2\sqrt[3]{x^2}}$ is defined for all $x \neq 0$. The complete set of values of x for which the function is decreasing is

(A) $x \leq -4, x > 0$

(B) $-1 \leq x < 0$

(C) $x \leq 2$

(D) $x \geq 2$

(E) $-1 \leq x \leq 2$

(F) $x \leq -1, x \geq 2$

5) Find the complete set of values of p for which the equation

$$x^2 - 3px + y^2 - 4y - p^2 + 6p + 7 = 0$$

describes a circle in the xy-plane.

(A) $p < -\frac{7}{3}$

(B) $p < \frac{12-10\sqrt{5}}{13}$ or $p > \frac{12+10\sqrt{5}}{13}$

(C) $p < \frac{10-10\sqrt{3}}{13}$ or $p > \frac{10+10\sqrt{3}}{13}$

(D) $p < \frac{12-10\sqrt{3}}{13}$ or $p > \frac{12+10\sqrt{3}}{13}$

(E) $p < -2$ or $p > 8$

(F) All real values of p

6) Find the finite area enclosed between the line $y = 0$ and the curve $y = x^2 - 5|x| - 24$.

(A) $\frac{544}{3}$

(B) $\frac{264}{3}$

(C) $\frac{1088}{3}$

(D) 162

(E) 216

(F) 432

7) For any real number x, let $\lfloor x \rfloor$ denote the greatest integer m such that $m \leq x$. Evaluate the following integral

$$\int_{-2}^{2} \lfloor x^2 - 1 \rfloor \, dx$$

(A) $2(3 - \sqrt{3} - \sqrt{2})$

(B) $2(5 - \sqrt{3} - \sqrt{2})$

(C) $2(1 - \sqrt{3} - \sqrt{2})$

(D) $3(5 - \sqrt{3} - \sqrt{2})$

(E) $3(1 - \sqrt{5} - \sqrt{3})$

(F) None of the above

8) This question is about pairs of functions f and g that satisfy

$$f(x) - g(x) = 3\sin x$$

$$f(x)g(x) = \cos^2 x$$

for all real numbers x. Across all solutions for $f(x)$, what is the minimum value that $f(x)$ attains for any x?

(A) $1 - \sqrt{3}$

(B) $-1 - \sqrt{3}$

(C) 0

(D) -1

(E) -2

(F) -3

(G) -4

9) This sequence of transformations is applied to the curve $y = 3x^2$:

1. Translation by $\begin{pmatrix} 2 \\ -4 \end{pmatrix}$
2. Reflection in the y-axis
3. Stretch parallel to the y-axis with scale factor 3

What is the equation of the resulting curve?

(A) $y = -3x^2 + 8x - 12$

(B) $y = -3x^2 + 8x - 6$

(C) $y = 9x^2 - 16x + 12$

(D) $y = 9x^2 - 36x + 24$

(E) $y = -27x^2 + 24x - 36$

(F) $y = -27x^2 + 24x - 18$

10) Given that

$$\left(a^3 + \frac{3}{b^3}\right)\left(\frac{3}{a^3} - b^3\right) = 8$$

where a and b are real numbers, what is the least value of ab?

(A) $-\sqrt{3}$

(B) $\sqrt{3}$

(C) $-2\sqrt{3}$

(D) $2\sqrt{3}$

(E) $-9^{\frac{1}{3}}$

(F) $-9^{\frac{1}{3}}$

(G) 1

(H) -1

11) A hollow right circular cone rests on a sphere. The height of the cone is 4 metres and the radius of the base is 1 metre. The volume of the sphere is the same as that of the cone. Then, the distance between the centre of the sphere and the vertex of the cone is

(A) 4 metres

(B) $\sqrt{17}$ metres

(C) $\sqrt{15}$ metres

(D) 5 metres

(E) 6 metres

(F) $\sqrt{13}$ metres

12) In the range $0 \leq x < 2\pi$, the equation

$$4\sin^2 x + 4\cos x = \frac{9}{2}$$

(A) has no solutions

(B) has 1 solution

(C) has 2 solutions

(D) has 3 solutions

(E) has 4 solutions

(F) has 5 solutions

13) Consider a rectangular cardboard box of height 3, breadth 4, and length 10 units. There is a lizard in one corner A of the box and an insect in the corner B, which is farthest from A. The length of the shortest path between the lizard and the insect along the surface of the box is

(A) $\sqrt{5^2 + 10^2}$ units

(B) $\sqrt{7^2 + 10^2}$ units

(C) $4 + \sqrt{3^2 + 10^2}$ units

(D) $3 + \sqrt{10^2 + 4^2}$ units

(E) $5 + \sqrt{10^2 + 7^2}$ units

(F) $6 + \sqrt{10^2 + 5^2}$ units

14) The equation in x

$$8x^4 - 16x^3 + 8x^2 + k = 0$$

has four real solutions:

(a) when $-30 < k < 4$

(b) when $4 < k < 30$

(c) when $-2.5 < k < 0$

(d) when $0 < k < 2.5$

(e) when $0 < k < 0.5$

(f) when $-0.5 < k < 0$

15) The number of pairs of positive integers x, y which solve the equation

$$x^3 + 6x^2y + 12xy^2 + 8y^3 = 2^{27}$$

is

(A) 0

(B) 2^5

(C) $2^8 - 1$

(D) $2^9 + 2$

(E) 2^7

(F) $2^6 - 1$

(G) $2^8 + 1$

16) Evaluate the following integral

$$\int_0^{100} e^{x-\lfloor x \rfloor}\, dx$$

(A) $\frac{e^{100}-1}{100}$

(B) $\frac{e^{100}-1}{e-1}$

(C) $100(e-1)$

(D) $\frac{e-1}{100}$

(E) $\frac{1-e}{100}$

(F) $\frac{e^{100}}{100}$

17) Consider six players P_1, P_2, P_3, P_4, P_5, and P_6. A team consists of two players (Thus, there are 15 distinct teams). Two teams play a match exactly once if there is no common player. For example, team $\{P_1, P_2\}$ cannot play with $\{P_2, P_3\}$, but will play with $\{P_4, P_5\}$. Then the total number of possible matches is

(A) 36

(B) 40

(C) 45

(D) 54

(E) 55

(F) 60

18) If $(\log_3 x)(\log_4 x)(\log_5 x) = (\log_3 x)(\log_4 x)+(\log_4 x)(\log_5 x)+(\log_5 x)(\log_3 x)$ and $x \neq 1$, then x is

(A) 10

(B) 100

(C) 50

(D) 60

(E) 80

(F) 90

19) The smallest value of

$$I(a) = \int_0^1 (x^2 - 2a)^2 dx,$$

as a varies, is

(A) $\frac{5}{21}$

(B) $\frac{6}{41}$

(C) $\frac{8}{23}$

(D) 1

(E) $\frac{9}{22}$

(F) $\frac{4}{45}$

20) Let

$$f(x) = (\tan x)^{3/2} - 3\tan x + \sqrt{\tan x}$$

Consider the three integrals

$$I_1 = \int_0^1 f(x)dx, \quad I_2 = \int_{0.3}^{1.3} f(x)dx, \quad I_3 = \int_{0.5}^{1.5} f(x)dx$$

Then,

(A) $I_1 > I_2 > I_3$

(B) $I_2 > I_1 > I_3$

(C) $I_3 > I_1 > I_2$

(D) $I_1 > I_3 > I_2$

(E) $I_2 > I_3 > I_1$

(F) $I_3 > I_2 > I_1$