Beyond Horizon TMUA Mock Paper

Set 1 Paper 2

75 Minutes

20 Questions

One answer per question

1) The complete set of values of x for which $(x^2-1)(x-2)>0$ is:

(A) $x<-1, 1<x<2$

(B) $x<-1, x>2$

(C) $-1<x<2$

(D) $x<1, x>2$

(E) $-1<x<1, x>2$

2) If m men can do a job in d days, then the number of days in which $m+r$ men can do the job is:

(A) $d+r$

(B) $\frac{d}{m+r}$

(C) $\frac{d}{m-r}$

(D) $\frac{md}{m+r}$

3) A club with x members is organized into four committees according to the following rules:

(i) Each member belongs to exactly two committees.

(ii) Each pair of committees has exactly one member in common.

Then:

(A) $x = 4$

(B) $x = 6$

(C) $x = 8$

(D) x cannot be determined from the given information.

4) Consider the relation "a person x shakes hand with a person y." If x shakes hand with y, then y shakes hand with x. In a gathering of 99 persons, one of the following statements is always true, considering 0 to be an even number. Which one is it?

(A) There is at least one person who shakes hand exactly with an odd number of persons.

(B) There is at least one person who shakes hand exactly with an even number of persons.

(C) There are even numbers of persons who shake hands exactly with an even number of persons.

(D) None of the foregoing statements.

5) Consider the sequence: $a_1 = 101, a_2 = 10101, a_3 = 1010101, \ldots$. Then a_k is a composite number (that is, not a prime number):

(A) if and only if $k \geq 2$ and 11 divides $10^{k+1} + 1$;

(B) if and only if $k \geq 2$ and 11 divides $10^{k+1} - 1$;

(C) if and only if $k \geq 2$ and $k - 2$ is divisible by 3;

(D) if and only if $k \geq 2$.

6) If the product of an odd number of odd integers is of the form $4n + 1$, then:

(A) an even number of them must always be of the form $4n + 1$;

(B) an odd number of them must always be of the form $4n + 3$;

(C) an odd number of them must always be of the form $4n + 1$;

(D) none of the above statements is true.

7) Let p, q, s be integers such that $p^2 = sq^2$. Then it follows that:

(A) p is an even number;

(B) if s divides p, then s is a perfect square;

(C) s divides p;

(D) q^2 divides p.

8) Given that $\log_p x = \alpha$ and $\log_q x = \beta$, the value of $\log_{p/q} x$ equals:

(A) $\frac{\alpha\beta}{\beta-\alpha}$;

(B) $\frac{\beta-\alpha}{\alpha\beta}$;

(C) $\frac{\alpha-\beta}{\alpha\beta}$;

(D) $\frac{\alpha\beta}{\alpha-\beta}$.

9) Let a, b, c be real numbers. Then the fourth-degree polynomial in x:

$$acx^4 + b(a+c)x^3 + (a^2+b^2+c^2)x^2 + b(a+c)x + ac$$

(A) has four complex (non-real) roots;

(B) has either four real roots or four complex roots;

(C) has two real roots and two complex roots;

(D) has four real roots.

10) Let S be the set of all numbers of the form $4^n - 3n - 1$, where $n = 1, 2, 3, \ldots$. Let T be the set of all numbers of the form $9(n-1)$, where $n = 1, 2, 3, \ldots$. Only one of the following statements is correct. Which one is it?

(A) Each number in S is also in T.

(B) Each number in T is also in S.

(C) Every number in S is in T and every number in T is in S.

(D) There are numbers in S which are not in T and there are numbers in T which are not in S.

11) Mr. Earl E. Bird leaves home every day at 8:00 AM to go to work. If he drives at an average speed of 40 miles per hour, he will be late by 3 minutes. If he drives at an average speed of 60 miles per hour, he will be early by 3 minutes. How many miles per hour does Mr. Bird need to drive to get to work exactly on time?

(A) 45

(B) 48

(C) 50

(D) 55

(E) 58

12) Define $x \heartsuit y$ to be $|x - y|$ for all real numbers x and y. Which of the following statements is not true?

(A) $x \heartsuit y = y \heartsuit x$ for all x and y

(B) $2(x \heartsuit y) = (2x) \heartsuit (2y)$ for all x and y

(C) $x \heartsuit 0 = x$ for all x

(D) $x \heartsuit x = 0$ for all x

(E) $x \heartsuit y > 0$ if $x \neq y$

13) A set of tiles numbered 1 through 100 is modified repeatedly by the following operation: remove all tiles numbered with a perfect square and renumber the remaining tiles consecutively starting with 1. How many times must the operation be performed to reduce the number of tiles in the set to one?

(A) 10

(B) 11

(C) 18

(D) 19

(E) 20

14) Evaluate the integral:

$$\int_{-0.5}^{2.5} 2^{\lfloor x \rfloor}\, dx$$

where $\lfloor x \rfloor$ is the floor of x.

(A) 5

(B) $\frac{13}{4}$

(C) $\frac{1}{4}$

(D) $\frac{15}{2}$

(E) $\frac{21}{4}$

15) When 15 is appended to a list of integers, the mean is increased by 2. When 1 is appended to the enlarged list, the mean of the enlarged list is decreased by 1. How many integers were in the original list?

(A) 4

(B) 5

(C) 6

(D) 7

(E) 8

16) For how many integers n is $\frac{n}{20-n}$ the square of an integer?

(A) 1

(B) 2

(C) 3

(D) 4

(E) 10

17) The mean, median, unique mode, and range of a collection of eight integers are all equal to 8. The largest integer that can be an element of this collection is:

(A) 11

(B) 12

(C) 13

(D) 14

(E) 15

18) Consider the following statements:

(1) For all polygons, all angles being equal is not sufficient for the polygon to be regular.

(2) There exists some polygon such that being able to draw a circle around the polygon which touches all its corners is sufficient for the polygon to be regular.

(3) For all n (where n can take any value in $\{1, 2, 3, 4\}$), there exists a hexagon with n lines of symmetry.

Which one of the following is/are true?

(A) None

(B) 1

(C) 2

(D) 3

(E) 1 and 2

(F) 1 and 3

(G) 2 and 3

(H) 1, 2, and 3

19) Let x, y, z be real numbers. Then only one of the following statements is true. Which one is it?

(A) If $x < y$, then $xz < yz$ for all values of z.

(B) If $x < y$, then $\frac{x}{z} \leq \frac{y}{z}$ for all values of z.

(C) If $x < y$, then $(x + z) < (y + z)$ for all values of z.

(D) If $0 < x < y$, then $xz < yz$ for all values of z.

20) Evaluate the following integral (integration by parts is not needed)

$$\int_{\frac{1}{2}}^{1} \ln x \, dx$$

(A) $\frac{\ln 2}{2}$

(B) $\frac{\ln 2 - 1}{2}$

(C) $\frac{1 - \ln 2}{2}$

(D) $-\ln 2$

(E) $\frac{-\ln 2}{2}$