PART E Advanced Mathematics and Advanced Physics

73 Which one of the following is a simplification of

$$1-\left(\frac{3+\sqrt{3}}{6-2\sqrt{3}}\right)^2$$

A $-\frac{3}{4}$

B $\frac{3}{4}$

C $-\frac{3}{4}-\frac{\sqrt{3}}{7}$

D $\frac{3}{4}-\frac{\sqrt{3}}{7}$

E $-\frac{3}{4}-\sqrt{3}$

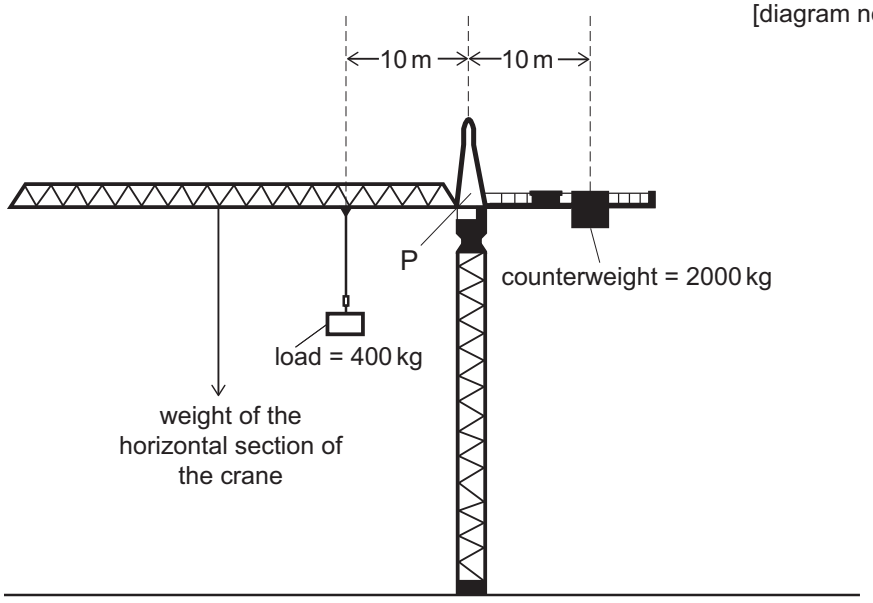
F $\frac{3}{4}-\sqrt{3}$

G $-\frac{\sqrt{3}}{2}$

H $\frac{\sqrt{3}}{2}$

74 The diagram shows a crane being used on a building site. The crane is perfectly balanced about P.

[diagram not to scale]



The load is now moved to the left by 5.0 m.

To keep the crane perfectly balanced about P, how far does the counterweight have to move, and in which direction?

(gravitational field strength = $10\,N\,kg^{-1}$)

A 1.0 m to the left

B 1.0 m to the right

C 3.0 m to the left

D 3.0 m to the right

E 4.0 m to the left

F 4.0 m to the right

75 k is the smallest positive value of x which is a solution to **both** the equations $2\sin x + 1 = 0$ **and** $2\cos 2x = 1$

How many values of x in the range $0 \leq x \leq k$ are solutions to at least one of these equations?

A 0

B 2

C 3

D 4

E 8

76 An aircraft is climbing at constant speed in a straight line at an angle of 10° to the horizontal.

Which statement about the resultant force on the aircraft is correct?

A It is parallel to its motion.

B It is perpendicular to its motion.

C It is zero.

D It is equal to its weight.

E It is equal to the drag acting on the aircraft.

77 Which of the following is a solution to the equation $3^{(2x+1)} - 6(3^x) = 0$?

A $\log_2 3$

B $\log_3 2$

C 2

D $\log_{10} 2$

E $\frac{2}{3}$

78 A ball starts at a speed of $40.0\,\text{m s}^{-1}$. The ball is subject to a constant deceleration of $14.4\,\text{m s}^{-2}$ as it travels a distance of 20.0 m in a straight line.

What is the final speed of the ball?

A $16.0\,\text{m s}^{-1}$

B $20.0\,\text{m s}^{-1}$

C $25.6\,\text{m s}^{-1}$

D $32.0\,\text{m s}^{-1}$

E $36.2\,\text{m s}^{-1}$

F $39.3\,\text{m s}^{-1}$

79 The graph of the function $y = x^3 + px^2 + qx + 6$, where p and q are real constants, has a local maximum when $x = 2$ and a local minimum when $x = 4$.

What are the values of p and q?

A $p = -3$ and $q = -8$

B $p = -3$ and $q = 8$

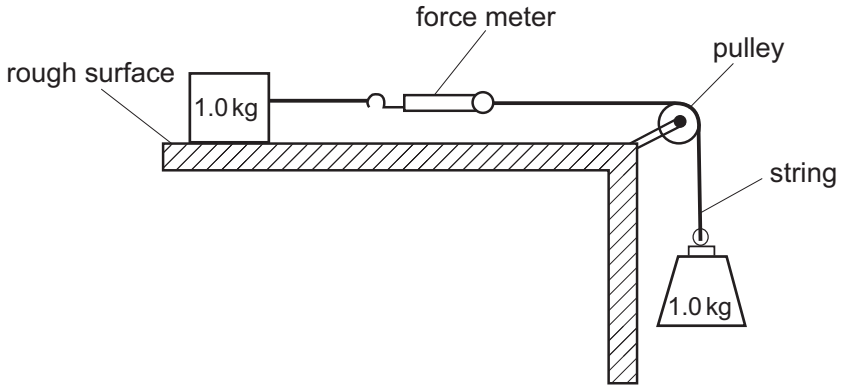
C $p = 3$ and $q = -8$

D $p = -9$ and $q = 24$

E $p = 9$ and $q = 24$

F $p = 9$ and $q = -24$

80 A block of mass 1.0 kg is at rest on a rough horizontal surface. The block is attached by a light inextensible string to a force meter. The other end of the force meter is attached by another light inextensible string via a frictionless pulley to a load of mass 1.0 kg. The block remains stationary.



What is the reading on the force meter?

(gravitational field strength = 10 N kg^{-1})

A 0.0 N

B 0.5 N

C 1.0 N

D 2.0 N

E 5.0 N

F 10 N

G 20 N

81 Given that $y = (2+3x)^6$, what is the coefficient of x^3 in $\frac{\mathrm{d}y}{\mathrm{d}x}$?

A 240

B 4320

C 4860

D 12 960

E 19 440

82 An apple of mass 100 g, growing on a tree, falls vertically from a height of 4.0 m above the ground. It hits the ground with a speed of $8.0\,\text{m}\,\text{s}^{-1}$.

How much work does the apple do against resistive forces during its descent, before it hits the ground?

(gravitational field strength $= 10\,\text{N}\,\text{kg}^{-1}$)

A 0.80 J

B 3.6 J

C 4.0 J

D 7.2 J

E 8.0 J

83 A geometric progression has first term equal to 1 and common ratio $\frac{1}{2}\sin 2x$

The sum to infinity of the series is $\frac{4}{3}$

Find the possible values of x in the range $\pi \leq x \leq 2\pi$

A $\frac{13}{12}\pi$, $\frac{17}{12}\pi$

B $\frac{7}{6}\pi$, $\frac{4}{3}\pi$

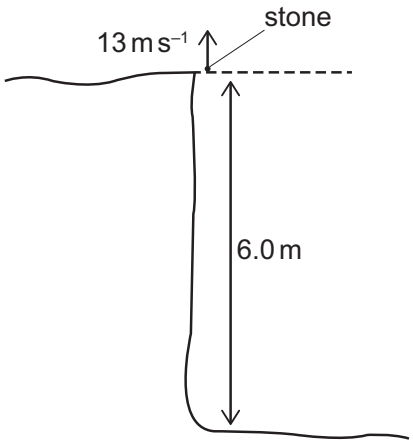
C $\frac{7}{6}\pi$, $\frac{11}{6}\pi$

D $\frac{5}{4}\pi$, $\frac{7}{4}\pi$

E there are no values of x in this range

84 A stone is fired vertically upwards at a speed of $13\,m\,s^{-1}$ on a still day from the top of a 6.0 m high cliff. It then falls down and lands at the bottom of the cliff.

[diagram not to scale]



From when the stone passes the top of the cliff on the way down, how long does it take to reach the ground at the bottom of the cliff?

(air resistance can be ignored; gravitational field strength = $10\,N\,kg^{-1}$)

A 0.40 s

B $\frac{6.0}{6.5}$ s

C 0.60 s

D $\sqrt{1.2}$ s

E 1.3 s

F 2.0 s

G 2.5 s

H 3.0 s

85 The sequence of numbers $u_1, u_2, u_3, \ldots, u_n, \ldots$ is given by

$$u_1 = 2$$
$$u_{n+1} = pu_n + 3$$

where p is an integer.

The fourth term, u_4, is equal to -7

What is the value of $u_1 + u_2 + u_3 + u_4$?

A -10

B -2

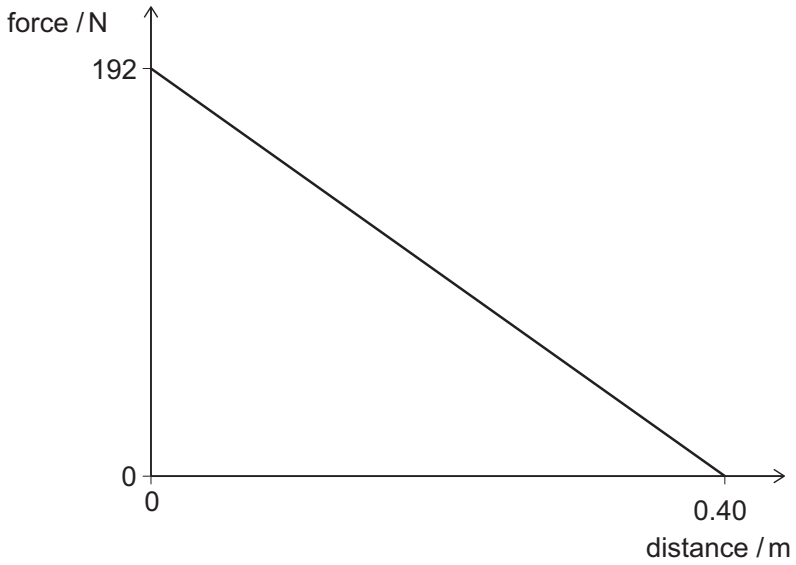
C -1

D 8

E 26

86 An archer fires an arrow of mass 0.024 kg vertically upwards from a bow.

The graph shows how the force of the bowstring on the arrow varies with distance as the arrow moves upwards.



The work done by the force of the bowstring is given by the area under the force-distance graph.

When the arrow leaves the bow, what is the kinetic energy of the arrow, and what is the maximum height that it gains from this point?

(Air resistance can be ignored. The effect of gravity as the arrow is fired is negligible compared to the force of the bowstring. The gravitational field strength = 10 N kg^{-1}.)

	kinetic energy / J	*height* / m
A	38.4	16
B	38.4	160
C	38.4	1600
D	38.4	16 000
E	76.8	32
F	76.8	320
G	76.8	3200
H	76.8	32 000

87 Find the complete set of values of x for which

$$\frac{x^3 - 6x^2 + 9x - 4}{x} > 0$$

A $x < 0,\ x > 4$

B $0 < x < 4$

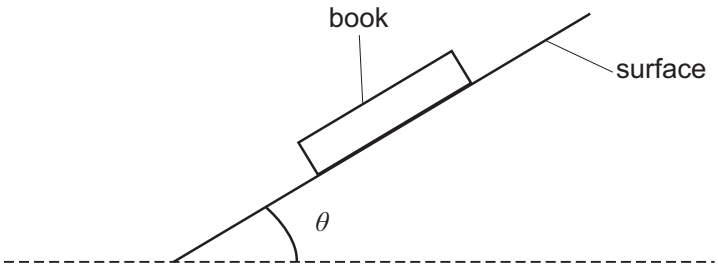
C $0 < x < 1,\ x > 4$

D $x < 0,\ 1 < x < 4$

E $x < 1,\ x > 4$

F $1 < x < 4$

88 A book of mass m rests on a rough horizontal surface. The surface is now tilted as shown:



When the angle of tilt θ is 20°, the book slides down the slope at a constant speed.

What is the acceleration of the book down the slope when the angle of tilt is 25°?

(gravitational field strength = g)

A $g(\cos 20° - \sin 20° \tan 5°)$

B $g(\cos 20° - \sin 20° \tan 25°)$

C $g(\cos 25° - \sin 5° \tan 20°)$

D $g(\cos 25° - \sin 25° \tan 20°)$

E $g(\sin 20° - \cos 20° \tan 5°)$

F $g(\sin 20° - \cos 20° \tan 25°)$

G $g(\sin 25° - \cos 5° \tan 20°)$

H $g(\sin 25° - \cos 25° \tan 20°)$

89 The equations of two straight lines are $y = 3 + (2p^2 - p)x$ and $y = 7 + (p - 2)x$, where p is a real constant.

For certain values of p, the two lines are perpendicular.

Which of the following numbers is closest to the greatest such value of p?

A 2.00

B 1.75

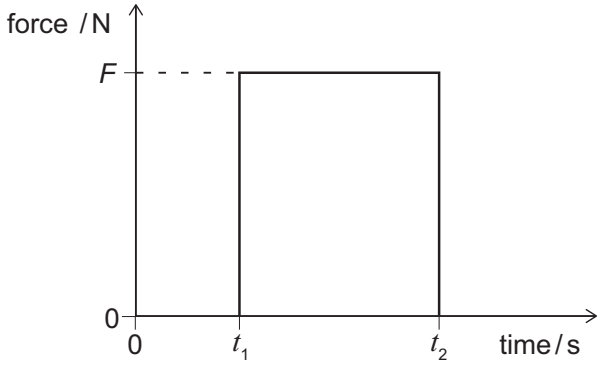
C 1.50

D 1.00

E -0.25

F -0.50

90 The graph shows how the horizontal force on a tennis ball of mass m varies during a shot in a tennis match. The ball is initially travelling horizontally toward the racket with speed u and leaves the racket horizontally travelling in the opposite direction with speed v.



Which expression gives the magnitude of the momentum of the ball as it leaves the racket?

A $F(t_2 - t_1)$

B $F(t_2 - t_1) - mu$

C $F(t_2 - t_1) + mu$

D $mv - mu$

E $Ft_2 - mu$