UNIVERSITY OF CAMBRIDGE

ECONOMICS ADMISSIONS ASSESSMENT

D563/11

November 2021

60 minutes

SECTION 1

INSTRUCTIONS TO CANDIDATES

Please read these instructions carefully, but do not open this question paper until you are told that you may do so. This paper is Section 1 of 2.

A separate answer sheet is provided for this paper. Please check you have one. You also require a soft pencil and an eraser.

Please complete the answer sheet with your candidate number, centre number, date of birth, and name.

At the end of 60 minutes, your supervisor will collect this question paper and answer sheet before giving out Section 2.

This paper contains **two** parts, **A** and **B**, and you should attempt **both** parts.

Part A Mathematics (20 questions)
Part B Advanced Mathematics (20 questions)

You are **strongly** advised to divide your time equally between the two parts: 30 minutes on **Part A** and 30 minutes on **Part B**. The scores for Part A and Part B are reported separately.

This paper contains 40 multiple-choice questions. There are no penalties for incorrect responses, only marks for correct answers, so you should attempt **all** 40 questions. Each question is worth one mark.

For each question, choose the **one** option you consider correct and record your choice on the separate answer sheet. If you make a mistake, erase thoroughly and try again.

You **must** complete the answer sheet within the time limit.

You can use the question paper for rough working, but **no extra paper** is allowed. Only your responses on the answer sheet will be marked.

Dictionaries and calculators are NOT permitted.

Please wait to be told you may begin before turning this page.

This question paper consists of 32 printed pages and 4 blank pages.

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PART A Mathematics

1 Simplify fully

$$5xy^2 \times (5x^2y)^{-3} \times 5x^2y$$

where x and y are positive.

A $\dfrac{1}{125x^7y^2}$

B $\dfrac{1}{125x^6y^2}$

C $\dfrac{1}{25x^6y}$

D $\dfrac{1}{25x^4y}$

E $\dfrac{1}{5x^3}$

F $\dfrac{1}{5x^2}$

G $\dfrac{y}{x^2}$

H $5xy^2$

2 Which of the following is a simplification of

$$2 - \frac{x + 3x^2}{12x^2 + x - 1}$$

where $x > 1$?

A $\dfrac{7x - 1}{4x - 1}$

B $\dfrac{7x - 2}{4x - 1}$

C $\dfrac{7x + 1}{4x + 1}$

D $\dfrac{7x + 2}{4x + 1}$

E $\dfrac{9x - 1}{4x - 1}$

F $\dfrac{9x - 2}{4x - 1}$

G $\dfrac{9x + 1}{4x + 1}$

H $\dfrac{9x + 2}{4x + 1}$

3 Which of the following is a rearrangement of

$$\frac{p}{2} + \frac{3}{q} = \frac{4}{r}$$

so that q is the subject?

A $q = \dfrac{2r}{24 - 3pr}$

B $q = \dfrac{3r}{2r - p}$

C $q = \dfrac{6r}{4 - p}$

D $q = \dfrac{6r}{8 - pr}$

E $q = \dfrac{r - 2}{12p}$

F $q = \dfrac{3r - 6}{4p}$

G $q = \dfrac{pr - 8}{12p}$

H $q = \dfrac{3pr - 24}{4p}$

4 A circle has its centre at $(0, 0)$.

What is the equation of the tangent that touches the circle at the point $(4, 3)$?

A $3y + 4x = 25$

B $3y - 4x = 25$

C $3y - 4x = -7$

D $3y - 4x = 7$

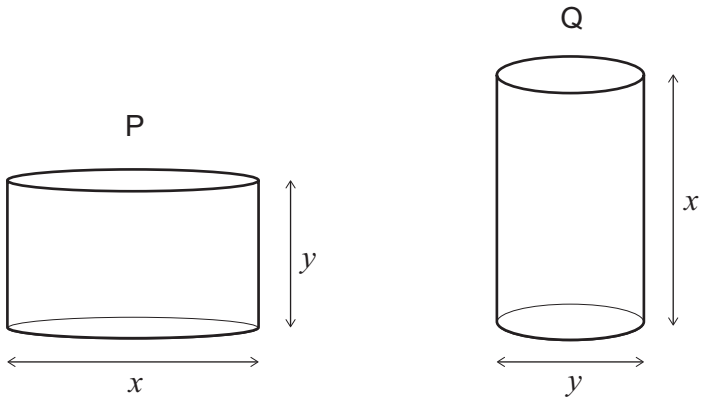
E $4y + 3x = 24$

F $4y - 3x = 24$

G $3y + 4x = 24$

H $3y - 4x = 24$

5 Two solid cylinders, P and Q, are shown, where $x > y$.



Cylinder P has diameter x and height y.

Cylinder Q has diameter y and height x.

What is the positive difference between the total surface areas of P and Q?

A 0

B $\frac{\pi}{4}(x^2 - y^2)$

C $\frac{\pi}{2}(x^2 - y^2)$

D $\pi(x^2 - y^2)$

E $2\pi(x^2 - y^2)$

F $\frac{\pi}{4}xy(x - y)$

G $\pi xy(x - y)$

6 Given that

$$8^x + 27^x = \frac{13}{36}$$

$$8^x - 27^x = \frac{5}{36}$$

what is the value of x?

A -4

B -3

C -2

D $-\frac{3}{2}$

E $-\frac{2}{3}$

F $-\frac{1}{2}$

G $-\frac{1}{3}$

H $-\frac{1}{4}$

7 The price of item P is reduced by 10%. The next day, the new price is increased by 10%.

The price of item Q is increased by 10%. The next day, the new price is reduced by 10%.

How does the final price of each item compare to the original price of that item?

	item P final price	*item Q final price*
A	lower than original	lower than original
B	lower than original	higher than original
C	higher than original	lower than original
D	higher than original	higher than original
E	the same as original	the same as original

8 Here is a pattern of numbers:

1						
2	3	4				
5	6	7	8	9		
10	11	12	13	14	15	16

The pattern of numbers is continued in the same way.

What number will appear directly below 196?

A 218

B 219

C 220

D 221

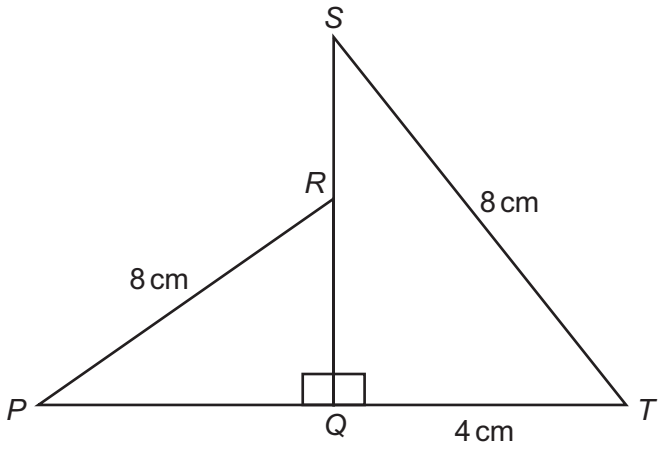
E 222

F 223

G 224

H 225

9



[diagram not to scale]

SQT is a right-angled triangle with the right angle at *Q*.

The point *R* is on *SQ* such that $SR : RQ = 1 : 3$

QRP is a right-angled triangle with the right angle at *Q*.

$PR = ST = 8\,\text{cm}$

$QT = 4\,\text{cm}$

What is the length of *PQ*, in cm?

A $2\sqrt{3}$

B $4\sqrt{3}$

C $\sqrt{19}$

D $\sqrt{37}$

E $\sqrt{55}$

F $\sqrt{61}$

10 Pat and Alex have a combined total of £63.

The ratio of Pat's money to Alex's money is 5 : 2

They each spend an equal amount on sweets.

The ratio of Pat's money to Alex's money is now 3 : 1

How much did Pat spend on sweets?

A £0.50

B £2.00

C £2.25

D £3.00

E £4.50

F £6.75

11 The curve with equation $y = x^2 - 4x + 5$ meets the straight line with equation $y = 2x + c$ at two points, which have x-coordinates p and q, where $q > p$.

Given that $q - p = 8$, what is the value of the constant c?

A -43

B -12

C -2

D 0

E 2

F 12

G 43

12 An online company sells storage containers.

The following items are available:

capacity of container	*number available*
2 litres	2
3 litres	3
7 litres	4
8 litres	1

A customer orders two containers at random from those available.

What is the probability that the two containers will have a combined capacity of exactly 10 litres?

A $\frac{7}{25}$

B $\frac{14}{25}$

C $\frac{7}{45}$

D $\frac{14}{45}$

E $\frac{7}{50}$

13 Given that

$$y = \frac{\sin 60° - 1}{\cos 60°}$$

what is the value of y^3?

A $-\frac{\sqrt{3}}{9}$

B $-5\sqrt{2}+10$

C $3\sqrt{3}-8$

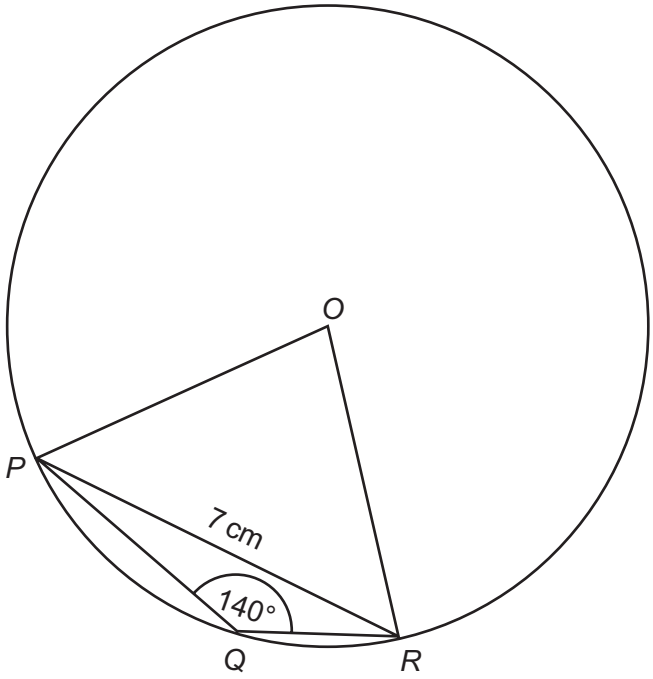
D $6\sqrt{3}-10$

E $14\sqrt{2}-20$

F $15\sqrt{3}-26$

G $21\sqrt{3}-38$

14 *P*, *Q* and *R* are points on the circumference of a circle with centre *O* as shown in the diagram.



[diagram not to scale]

Angle $PQR = 140°$

$PR = 7$ cm

Which of the following expressions gives the radius of the circle, in cm?

A $7 \sin 10°$

B $3.5 \sin 55°$

C $3.5 \sin 70°$

D $7 \sin 55°$

E $\dfrac{3.5}{\sin 40°}$

F $\dfrac{7}{\sin 80°}$

G $\dfrac{3.5}{\sin 20°}$

H $\dfrac{7}{\sin 40°}$

15 Charlie has a bowl containing red sweets and green sweets only. The sweets are identical in all respects except colour.

There are nine sweets in total in the bowl.

Charlie eats two sweets from the bowl at random.

The probability of Charlie not eating any green sweets is $\frac{5}{12}$

What is the probability that Charlie eats two green sweets?

A $\frac{2}{27}$

B $\frac{1}{12}$

C $\frac{1}{9}$

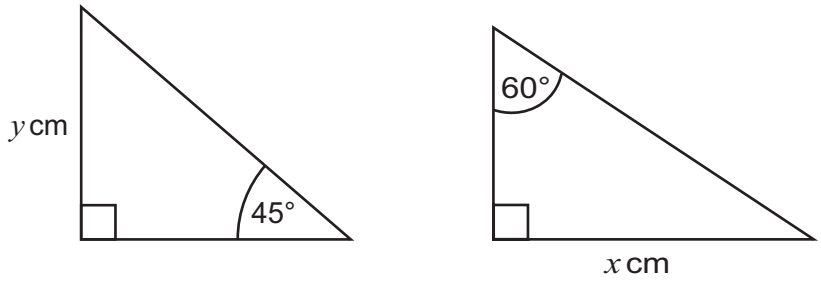
D $\frac{4}{27}$

E $\frac{1}{6}$

F $\frac{1}{4}$

G $\frac{7}{12}$

16 The following right-angled triangles have the same hypotenuse length.



[diagram not to scale]

Which of the following is a correct expression for y in terms of x?

A $y = \sqrt{2}\,x$

B $y = \dfrac{\sqrt{2}\,x}{2}$

C $y = \dfrac{\sqrt{2}\,x}{3}$

D $y = \dfrac{\sqrt{2}\,x}{6}$

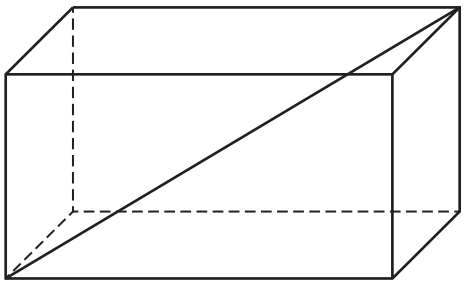
E $y = \sqrt{6}\,x$

F $y = \dfrac{\sqrt{6}\,x}{2}$

G $y = \dfrac{\sqrt{6}\,x}{3}$

H $y = \dfrac{\sqrt{6}\,x}{6}$

17 The greatest diagonal distance between the two vertices of a cuboid, as shown in the diagram, is $\sqrt{77}$ cm.



A similar cuboid has all its lengths exactly half the lengths of the original cuboid.

The sides of this smaller cuboid are 2 cm, 3 cm and x cm.

What is the value of x, in cm?

A $\frac{5}{2}$

B 5

C $\frac{5\sqrt{2}}{2}$

D $5\sqrt{2}$

E $\frac{\sqrt{102}}{2}$

F $\sqrt{102}$

18 Alex, Cameron and Sam are all taking part in a 400 m race.

They are each running at a different constant speed.

Alex is running 12% faster than Cameron, whilst Sam is running 2% slower than Cameron.

When Alex crosses the finish line, how many metres is Sam from the finish line?

A 9.6

B 14

C 24

D 25

E 28

F 50

G 56

19 A car journey is m miles long.

One kilometre is equivalent to x miles.

The car uses one litre of fuel to travel a distance of f kilometres.

Fuel for the car costs p pence per litre.

Which of the following expressions gives the cost of fuel for this journey, in pounds?

(There are 100 pence in one pound.)

A $100fmpx$

B $\frac{100fmp}{x}$

C $\frac{100mpx}{f}$

D $\frac{100mp}{fx}$

E $\frac{fmpx}{100}$

F $\frac{fmp}{100x}$

G $\frac{mpx}{100f}$

H $\frac{mp}{100fx}$

20 How many solutions are there to the equation

$$\tan x = 100x$$

where $-360 \leq x \leq 360$?

A 0

B 1

C 2

D 3

E 4

F 5

G infinitely many

PART B Advanced Mathematics

21 Given that

$$y = \left(2\sqrt{x} - \frac{1}{2\sqrt{x}}\right)^2$$

find the value of $\frac{dy}{dx}$ when $x = \frac{1}{2}$

A -12

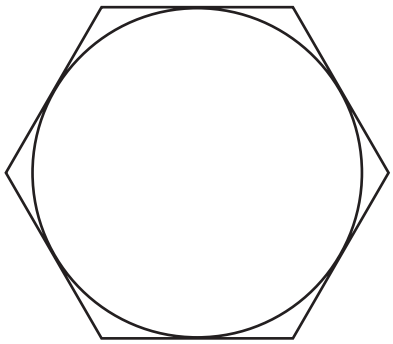
B $-\frac{1}{4}$

C 3

D $\frac{63}{16}$

E 5

22



The diagram shows a circle with radius 2 cm, and a regular hexagon drawn so that each of its edges is tangent to the circle.

What is the area of the hexagon, in cm^2?

A $4\sqrt{3}$

B $6\sqrt{3}$

C $8\sqrt{3}$

D $12\sqrt{3}$

E $24\sqrt{3}$

23 A particular arithmetic series has first term a and common difference d.

The sum of the first k terms of this series is denoted by S_k

Which of the following is a simplification of $S_{n+1} - S_{n-1}$?

A d

B $2d$

C $2a + d$

D $2a + 2d$

E $2a + nd$

F $2a + 2nd$

G $2a + (2n - 1)d$

H $2a + (4n - 2)d$

24 Evaluate

$$\frac{\left(2+\sqrt{3}\right)^4 - \left(2-\sqrt{3}\right)^4}{8}$$

A 16

B 18

C 22

D $2\sqrt{3}$

E $8\sqrt{3}$

F $14\sqrt{3}$

G $17\sqrt{3}$

H $56\sqrt{3}$

25 Find how many distinct real solutions there are to the equation

$$(x^2 + 4x + 3)^2 = 1$$

A 0

B 1

C 2

D 3

E 4

26 The polynomial $x^2 - x - 1$ is a factor of the polynomial $px^3 + qx^2 - 1$ where p and q are constants.

What is the value of q?

A 2

B 1

C 0

D -1

E -2

27 The line $x = 1$ divides the circle $x^2 + y^2 = 4$ into two segments.

What is the area of the smaller segment?

A $\frac{2\pi}{3} - \frac{\sqrt{3}}{2}$

B $\frac{2\pi}{3} - \sqrt{3}$

C $\frac{\pi}{2} - \frac{1}{2}$

D $\frac{\pi}{2} - 1$

E $\pi - \frac{1}{2}$

F $\pi - 1$

G $\frac{4\pi}{3} - \frac{\sqrt{3}}{2}$

H $\frac{4\pi}{3} - \sqrt{3}$

28 The quadratic function $f(x)$ has remainder 3 when divided by $(x - 1)$.

$f(x)$ has remainder 5 when divided by $(x + 3)$.

One solution of the equation $f(x) = 0$ is $x = 2$

What is the coefficient of x in $f(x)$?

A $-\frac{1}{2}$

B $-\frac{3}{2}$

C $-\frac{5}{2}$

D -4

E -9

29 What is the mean of $\log_{10} 27$, $\log_{10} 64$, and $\log_{10} 216$?

A $\dfrac{\log_{10} 307}{3}$

B $\dfrac{\log_{10} 81}{3}$

C $\dfrac{\log_{10} 6^{12}}{3}$

D $\log_{10} 64$

E $\log_{10} 72$

F $\log_{10} 108$

30 The line $y = \dfrac{1}{2}x + c$ meets the curve $y = \dfrac{1}{8}x^2$ at two points, *P* and *Q*.

The midpoint of the line segment *PQ* has y-coordinate 5.

What is the value of c?

A 0

B $\dfrac{3}{2}$

C 3

D 4

E $\dfrac{9}{2}$

F 5

G 6

31 Which of the following is the largest in value?

(All angles are in radians.)

A $\cos 0.5$

B $\cos 0.75$

C $\cos 1$

D $\sin 0.5$

E $\sin 0.75$

F $\sin 1$

32 The circles

$$x^2 + 2x + y^2 = 19$$

and

$$x^2 - 2x + y^2 - 2y = 3$$

intersect only at the point (p, q).

What is the value of $p + q$?

A –11

B –7

C –5

D –3

E 3

F 5

G 7

H 11

33 A geometric progression has first term $u_1 = a$ and common ratio r.

The sum to infinity of the geometric progression is $\frac{8}{5}$

The sum to infinity of the even-numbered terms ($u_2 + u_4 + u_6 + \cdots$) is $\frac{3}{5}$

What is the value of $a + r$?

A $\frac{3}{5}$

B $\frac{31}{25}$

C $\frac{23}{5}$

D $\frac{28}{5}$

E $\frac{67}{8}$

34 What is the complete set of possible values of k for which the graphs of $y = k$ and $y = x|4 - x|$ have exactly two distinct points of intersection?

A $k < -4$, $k = 0$

B $k = -4$, 0

C $k > 4$

D $0 < k < 4$

E $k = 0$, 4

F $k < 4$

G $k = 0$, $k > 4$

35 At how many distinct points do the following two curves meet?

$$y = (x - 4)(x^2 - 2x - 8)$$

$$y = -x^2 + 8x - 16$$

A 0

B 1

C 2

D 3

E 4

F 5

36 Find the complete set of real values of k for which the equation $3^{x+1} + 3^{-x} = k$ has at least one real root for x.

A $k \geq 0$

B $k \geq 2$

C $k \geq 3$

D $k \geq 2\sqrt{3}$

E $k \geq 2$ or $k \leq -2$

F $k \geq 2\sqrt{3}$ or $k \leq -2\sqrt{3}$

G $k \geq \log_3 2$ or $k \leq -\log_3 2$

37 Evaluate

$$\frac{3}{\sqrt{27}+\sqrt{21}} + \frac{3}{\sqrt{24}+\sqrt{18}} + \frac{3}{\sqrt{21}+\sqrt{15}} + \cdots + \frac{3}{\sqrt{9}+\sqrt{3}}$$

A $\frac{3\sqrt{2}}{2}$

B $3\sqrt{2}$

C $\frac{3\sqrt{3}}{2}$

D $\sqrt{3}$

E $1+\sqrt{2}$

F $3\left(1+\sqrt{2}\right)$

G $\frac{\sqrt{3}}{3}\left(1+\frac{\sqrt{2}}{2}\right)$

H $\sqrt{3}\left(1+\frac{\sqrt{2}}{2}\right)$

38 Find the complete set of values of θ for which the following inequality is valid:

$$\sin\theta < \int_{\sin\theta}^{\cos\theta} (2x-1)\,\mathrm{d}x \qquad \text{where } 0 < \theta < 2\pi$$

A $0 < \theta < 2\pi$

B $\frac{\pi}{6} < \theta < \frac{11\pi}{6}$

C $\frac{\pi}{3} < \theta < \frac{5\pi}{3}$

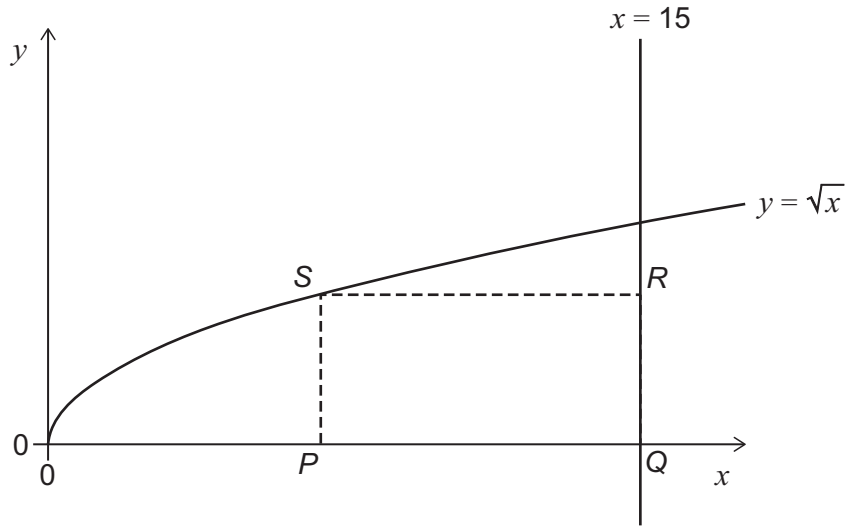
D $\frac{2\pi}{3} < \theta < \frac{4\pi}{3}$

E $\frac{5\pi}{6} < \theta < \frac{7\pi}{6}$

F $\frac{\pi}{4} < \theta < \frac{3\pi}{4}, \ \frac{5\pi}{4} < \theta < \frac{7\pi}{4}$

G $0 < \theta < \frac{\pi}{4}, \ \frac{3\pi}{4} < \theta < \frac{5\pi}{4}, \ \frac{7\pi}{4} < \theta < 2\pi$

39



PQRS is a rectangle.

P and *Q* lie on the x-axis.

Q and *R* lie on the line $x = 15$

S lies on the curve $y = \sqrt{x}$

What is the maximum possible area of the rectangle?

A $5\sqrt{5}$

B $10\sqrt{5}$

C 50

D $25\sqrt{5}$

E 100

F 125

40 Find the number of solutions to the equation

$$\Big|\,\big|\,|x-1|-1\big|-1\Big|-1=0$$

A 0

B 1

C 2

D 3

E 4

F 5

G 6

END OF TEST

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