UNIVERSITY OF CAMBRIDGE

ECONOMICS **D563/31**
ADMISSIONS ASSESSMENT

November 2020 **60 minutes**

SECTION 1

* 8 7 2 9 6 4 0 2 7 4 *

INSTRUCTIONS TO CANDIDATES

Please read these instructions carefully, but do not open this question paper until you are told that you may do so. This paper is Section 1 of 2.

A separate answer sheet is provided for this paper. Please check you have one. You also require a soft pencil and an eraser.

Please complete the answer sheet with your candidate number, centre number, date of birth, and name.

At the end of 60 minutes, your supervisor will collect this question paper and answer sheet before giving out Section 2.

This paper contains **two** parts, **A** and **B**, and you should attempt **both** parts.

Part A Mathematics (20 questions)
Part B Advanced Mathematics (20 questions)

You are **strongly** advised to divide your time equally between the two parts: 30 minutes on **Part A** and 30 minutes on **Part B**. The scores for Part A and Part B are reported separately.

This paper contains 40 multiple-choice questions. There are no penalties for incorrect responses, only marks for correct answers, so you should attempt **all** 40 questions. Each question is worth one mark.

For each question, choose the **one** option you consider correct and record your choice on the separate answer sheet. If you make a mistake, erase thoroughly and try again.

You **must** complete the answer sheet within the time limit.

You can use the question paper for rough working, but **no extra paper** is allowed. Only your responses on the answer sheet will be marked.

Dictionaries and calculators are NOT permitted.

Please wait to be told you may begin before turning this page.

This question paper consists of 27 printed pages and 1 blank page.

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PART A Mathematics

1 The admission charge to a cinema is different for adults and children.

Admission for 2 adults and 3 children costs £20.

Admission for 4 adults and 4 children costs £34.

What does admission cost for 6 adults and 2 children?

A £27

B £29

C £33

D £39

E £44

F £48

G £72

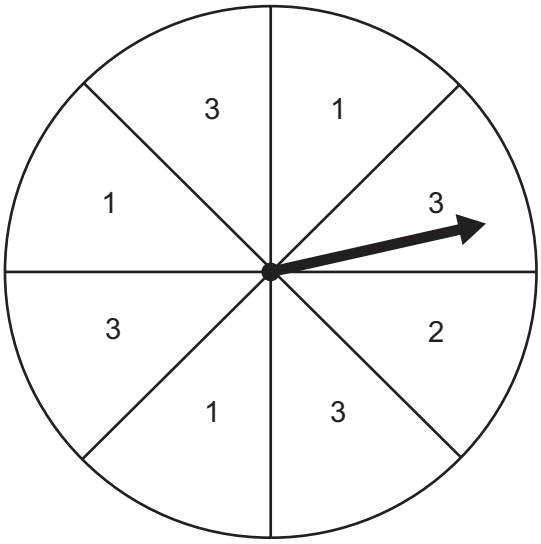
2 The n^{th} term of a sequence is $2n - 5$.

Which row in the table is correct for this sequence?

	term-to-term rule	*term which has a value of 17*
A	subtract 5	11^{th}
B	subtract 5	29^{th}
C	subtract 2	11^{th}
D	subtract 2	29^{th}
E	add 5	11^{th}
F	add 5	29^{th}
G	add 2	11^{th}
H	add 2	29^{th}

3 A fair spinner has eight equal sections.

Each section has one number written on it, as shown.



The spinner is spun twice, and the two numbers scored are added.

What is the probability that the sum of the two numbers is 5?

A $\frac{1}{8}$

B $\frac{5}{8}$

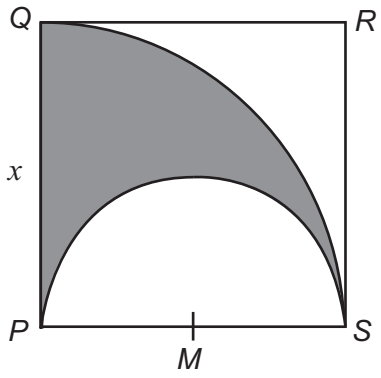
C $\frac{1}{16}$

D $\frac{3}{16}$

E $\frac{25}{64}$

F $\frac{55}{64}$

4



PQRS is a square with side length x.

M is the midpoint of side *PS*.

A circular arc, with centre *M*, is drawn inside the square from *S* to *P*.

Another circular arc, with centre *P*, is drawn inside the square from *S* to *Q*.

What is the area of the shaded region in terms of x?

A $\frac{1}{8}\pi x^2$

B $\frac{3}{16}\pi x^2$

C $\frac{1}{4}\pi x^2$

D $\frac{5}{16}\pi x^2$

E $\frac{3}{8}\pi x^2$

F $\frac{7}{16}\pi x^2$

G $\frac{1}{2}\pi x^2$

5 A balloon contains 5000 cm^3 of gas.

The gas in the balloon gradually escapes so that the volume of the balloon decreases.

60% of the volume of the balloon is lost each week.

What is the volume of the balloon, in cm^3, after 3 weeks?

A 0

B 128

C 320

D 800

E 1080

6 Consider the four lines with the following equations.

1 $2x + 6y = 3$

2 $9y = 3x - 4$

3 $2y = 6x + 3$

4 $4x + 6y - 9 = 0$

Which two lines are perpendicular?

A 1 and 2

B 1 and 3

C 1 and 4

D 2 and 3

E 2 and 4

F 3 and 4

7 The equilateral triangle *PQR* has sides of length 8 cm.

A circle, centre *O*, passes through each of the vertices of the triangle.

Find an expression for the circumference of the circle, in cm.

A $\dfrac{\sin 60°}{8\pi}$

B $\dfrac{8\pi}{\sin 60°}$

C $\dfrac{\cos 60°}{8\pi}$

D $\dfrac{8\pi}{\cos 60°}$

E $\dfrac{\tan 60°}{8\pi}$

F $\dfrac{8\pi}{\tan 60°}$

8 Find the sum of the solutions of

$$2\left(\frac{x}{4}+3\right)^2-\left(\frac{x}{4}+3\right)-36=0$$

A 2

B $\dfrac{3}{2}$

C $\dfrac{1}{2}$

D -4

E -13

F -22

G -26

H -34

9 When the expression

$$(2x + 3)^2 - (x - 3)^2$$

is written in the form $p(x + q)^2 + r$, where p, q and r are constants, what is the value of r?

A -27

B -9

C 0

D 3

E 15

10 Which one of the following expressions is equivalent to

$$\frac{a}{b/c} - \frac{a/b}{c}$$

A 0

B $\frac{a(b^2 - 1)}{bc}$

C $\frac{a(b^2 - c^2)}{bc}$

D $\frac{a^2b^2 - c^2}{abc}$

E $\frac{a(c^2 - 1)}{bc}$

F $\frac{a^2c^2 - b^2}{abc}$

G $\frac{b^2 - a^2}{abc}$

11 The table shows statistics relating to the test marks of two groups of students.

	number of students	*mean*	*range*
group X	10	36	16
group Y	20	48	21

The results for the two groups of students are combined.

What can be deduced about the mean and range of the combined results?

A mean = 40, range $\leq$ 16

B mean = 40, 16 < range < 21

C mean = 40, range $\geq$ 21

D mean = 44, range $\leq$ 16

E mean = 44, 16 < range < 21

F mean = 44, range $\geq$ 21

12 The number of pairs of winter boots sold on a day is inversely proportional to the cube of the outside temperature on that day, measured in °C.

On a day when the outside temperature is 8 °C, 250 pairs of boots are sold.

The next day, when the outside temperature is x °C, the number of pairs of boots sold is 700% more than on the previous day.

What is the value of x?

A 2

B 4

C $\frac{8}{\sqrt[3]{7}}$

D $8\sqrt[3]{7}$

E 16

13 In a sale, all prices are reduced by 25%.

A customer calculates the pre-sale price of a bicycle incorrectly by increasing the marked sale price by 25%.

The customer's calculated pre-sale price is incorrect by £15.

What is the correct pre-sale price of the bicycle?

A £180

B £195

C £210

D £225

E £240

14 A paint colour is a mixture of red paint, blue paint and yellow paint.

The ratio of red paint to blue paint in the mixture is 18 : 5

The ratio of blue paint to yellow paint in the mixture is p : 3

The ratio of red paint to yellow paint in the mixture is 12 : 5

What is the value of p?

A 2

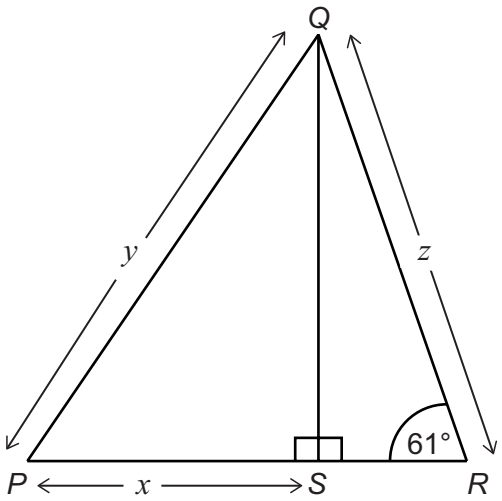
B 4.5

C 5

D 7.5

E 12

15



[diagram not to scale]

In the diagram, *QS* is perpendicular to *PR*.

$PS = x$ cm
$PQ = y$ cm
$QR = z$ cm

angle $QRS = 61°$

PSR is a straight line.

Which one of the following is an expression for the length z, in cm?

A $\sqrt{y^2 + x^2}\sin 61°$

B $\sqrt{y^2 - x^2}\sin 61°$

C $\sqrt{y^2 + x^2}\cos 61°$

D $\sqrt{y^2 - x^2}\cos 61°$

E $\dfrac{\sqrt{y^2 + x^2}}{\sin 61°}$

F $\dfrac{\sqrt{y^2 - x^2}}{\sin 61°}$

G $\dfrac{\sqrt{y^2 + x^2}}{\cos 61°}$

H $\dfrac{\sqrt{y^2 - x^2}}{\cos 61°}$

16 Two identical fair six-sided dice each have their faces numbered from 1 to 6, with one number on each face.

Both dice are thrown, and the number on each of the dice is recorded.

They are then both thrown again, and the number on each of the dice is recorded.

What is the probability that at least one of the four recorded numbers is even?

A $\frac{1}{4}$

B $\frac{1}{2}$

C $\frac{9}{16}$

D $\frac{3}{4}$

E $\frac{15}{16}$

17 The quadratic equation $2x^2 - px - 4 = 0$, where p is a positive constant, has two solutions that differ by 6.

What is the value of p?

A 2

B $4\sqrt{7}$

C 12

D $4\sqrt{11}$

E $4\sqrt{34}$

F $6\sqrt{30}$

18 Two vertices of a square are at $(1, 1)$ and $(3, 5)$.

What is the difference between the perimeters of the largest and smallest possible squares that can be drawn with these points as two of their vertices?

A 0

B $4\sqrt{3}\left(2-\sqrt{2}\right)$

C $4\sqrt{3}\left(\sqrt{2}-1\right)$

D $4\sqrt{5}\left(2-\sqrt{2}\right)$

E $4\sqrt{5}\left(\sqrt{2}-1\right)$

F $4\sqrt{13}\left(2-\sqrt{2}\right)$

G $4\sqrt{13}\left(\sqrt{2}-1\right)$

H $4\sqrt{3}\sqrt{5}\left(2-\sqrt{2}\right)$

19 The point M is $(2, 5)$ and the point N is $(-3, -1)$.

The line segment MN is transformed to the line segment TU by two transformations:

MN is rotated 90° clockwise about the origin to give the line segment RS.

RS is then translated by the vector $\begin{pmatrix} p \\ q \end{pmatrix}$ to give the line segment TU.

The coordinates of the midpoint of TU are $(7, -2.5)$.

Find the vector $\begin{pmatrix} p \\ q \end{pmatrix}$

A $\begin{pmatrix} 2 \\ 0.5 \end{pmatrix}$

B $\begin{pmatrix} 0.5 \\ 2 \end{pmatrix}$

C $\begin{pmatrix} 5 \\ -3 \end{pmatrix}$

D $\begin{pmatrix} -3 \\ 5 \end{pmatrix}$

E $\begin{pmatrix} 9 \\ -2 \end{pmatrix}$

F $\begin{pmatrix} -2 \\ 9 \end{pmatrix}$

20 A solid cone has a base radius x cm.

The ratio of the perpendicular height of the cone to the radius of the cone is 5 : 2

A solid hemisphere of radius $\frac{y}{2}$ cm is made from the same material as the cone.

Which one of the following is a correct expression for

$$\frac{\text{volume of the cone}}{\text{volume of the hemisphere}}$$

(Volume of a cone = $\frac{1}{3}\pi r^2 h$ where r is the radius and h is the perpendicular height.)

(Volume of a sphere = $\frac{4}{3}\pi r^3$ where r is the radius.)

A $\frac{5x^3}{y^3}$

B $\frac{5x^3}{4y^3}$

C $\frac{8x^3}{5y^3}$

D $\frac{10x^3}{y^3}$

E $\frac{14x^3}{y^3}$

PART B Advanced Mathematics

21 Given that

$$\int_4^9 f(x)\mathrm{d}x = 3$$

and that

$$g(x) = 2f(x) + 1$$

find

$$\int_4^9 g(x)\mathrm{d}x$$

A 6

B 7

C 8

D 9

E 10

F 11

22 $(x - 1)$ and $(x - 2)$ are both factors of $x^4 + ax^3 + bx^2 - 12x + 4$

What are the values of a and b?

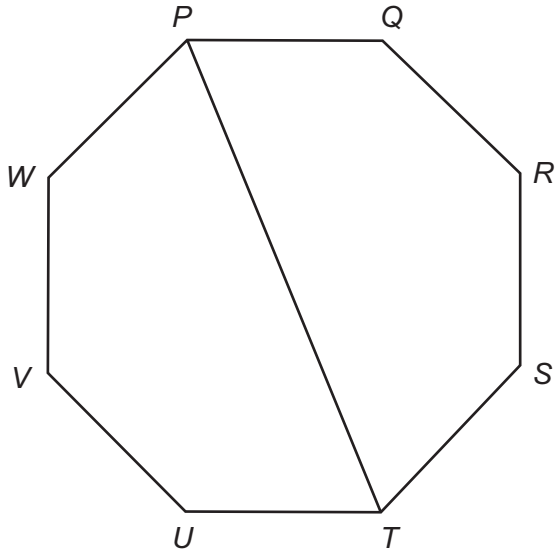
A $a = -6$ and $b = -23$

B $a = -6$ and $b = 13$

C $a = 6$ and $b = -11$

D $a = 6$ and $b = 1$

23



The area of the regular octagon is $32\sqrt{2}$ cm^2.

What is the length, in cm, of the straight line *PT*?

A 8

B 16

C $4\sqrt{2}$

D $8\sqrt{2}$

E $4\sqrt{2\sqrt{2}}$

F $16\sqrt{2\sqrt{2}}$

24 What is the area of the region enclosed between the curve $y = \frac{1}{2}x^2$, the line $y = -x$, and the lines $x = 1$ and $x = 3$?

A $\frac{1}{3}$

B 2

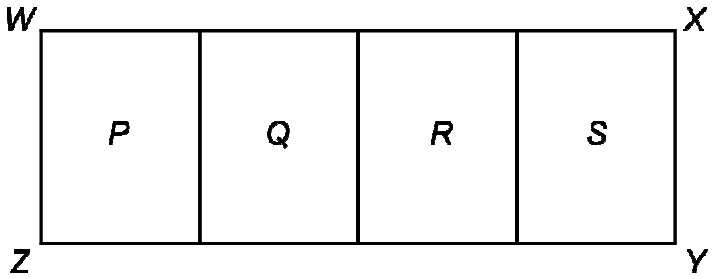
C 4

D 6

E $\frac{25}{3}$

F $\frac{28}{3}$

25



A wall is used to mark out the perimeter of a rectangular field *WXYZ*.

Walls are also used to divide *WXYZ* into four identical rectangular fields, *P*, *Q*, *R* and *S*, as shown.

The total length of wall used is 260 m.

What is the length of *WZ* that maximises the area *P*?

A 20 m

B 26 m

C 32.5 m

D 40 m

E 52 m

F 65 m

26 A line with non-zero gradient m is reflected in the line $y = x$

What is the gradient of the reflected line?

A m

B $-m$

C $\frac{1}{m}$

D $-\frac{1}{m}$

27 There are two red balls and two blue balls in a bag.

Two balls are removed at random without replacement.

Given that at least one of them is red, what is the probability that one of them is blue?

A $\frac{1}{2}$

B $\frac{2}{3}$

C $\frac{4}{5}$

D $\frac{5}{6}$

E 1

28 The sum of the first 20 terms of an arithmetic progression is 50.

The sum of the next 20 terms of the arithmetic progression is -50.

What is the sum of the first 100 terms of the arithmetic progression?

A -750

B -350

C -50

D $-\frac{159}{8}$

E $\frac{159}{8}$

F 50

G 350

H 750

29 A sequence is generated by

$$x_{n+1} = -\frac{12}{x_n}$$

n is an integer, where $n \geq 1$.

The 50th term of the sequence is 6.

What is the sum of the first fifteen terms of the sequence?

A -2

B 6

C 10

D 22

E 26

F 34

G 58

30 The line L with equation $y = mx + c$, where $m > 0$ and $c \geq 0$, passes through the point $(2, 4)$.

A line is drawn through the point $(2, 4)$ perpendicular to L.

The triangle enclosed between the two lines and the y-axis has area 5 square units.

What is the **larger** of the two possible values of m?

A -0.5

B 0.5

C 1.25

D 2

E 5

31 A list of n numbers has mean m and a unique mode d.

Two numbers are removed from the list.

The remaining list of numbers also has a unique mode, but this unique mode is not equal to d.

The mean of the remaining $n - 2$ numbers is $m + 2$.

What was the unique mode, d, of the original list?

A $n - m + 2$

B $n - m - 2$

C $n + m - 2$

D $m + n + 2$

E $m - n + 2$

F $m - n - 2$

32 *P* and *Q* are two different geometric progressions.

The 3rd term of each geometric progression is 4.

The 5th term of each geometric progression is 2.

What is the modulus of the difference between the sums to infinity of *P* and *Q*?

A 0

B 8

C $8\sqrt{2}$

D 16

E $16\sqrt{2}$

F 32

G $32\sqrt{2}$

33 Two circles have centres *P* and *Q*.

The radius of each circle is 1 cm.

The distance *PQ* is 1 cm.

What is the area of overlap, in cm^2, of the two circles?

A $\frac{\pi}{3} - \frac{1}{4}$

B $\frac{\pi}{3} - \frac{\sqrt{3}}{4}$

C $\frac{2\pi}{3} - \frac{1}{2}$

D $\frac{2\pi}{3} - \frac{\sqrt{3}}{2}$

E $\frac{4\pi}{3} - \frac{1}{4}$

F $\frac{4\pi}{3} - \frac{\sqrt{3}}{2}$

34 The curve

$$y = x^3 + 3\sqrt{5}px^2 + 3px + 13$$

has two distinct turning points.

What are all the possible values of p?

A $p < 0$, $p > 0.2$

B $p \leq 0$, $p \geq 0.2$

C $0 < p < 0.2$

D $0 \leq p \leq 0.2$

E $p < 0$, $p > 1.2$

F $p \leq 0$, $p \geq 1.2$

G $0 < p < 1.2$

H $0 \leq p \leq 1.2$

35 Find the complete set of values of x for which

$$x^4 + 36 < 13x^2$$

A $4 < x < 9$

B $x < 4$, $x > 9$

C $-9 < x < -4$, $4 < x < 9$

D $x < -9$, $-4 < x < 4$, $x > 9$

E $2 < x < 3$

F $x < 2$, $x > 3$

G $-3 < x < -2$, $2 < x < 3$

H $x < -3$, $-2 < x < 2$, $x > 3$

36 Find the number of solutions of the equation

$$14\cos^3 x + 10\sin^2 x \cos x = 13\cos x$$

in the range $-2\pi \leq x \leq 2\pi$

A 4

B 6

C 8

D 10

E 12

F 14

37 It is given that

$$2^x = 3^y$$

and

$$x + y = 2$$

Which one of the following is an expression for x?

A $2\log_{10}\frac{1}{2}$

B $\log_{10} 3$

C $\frac{6}{5}$

D $\frac{3}{2}$

E $\frac{\log_{10} 9}{\log_{10} 5}$

F $\frac{\log_{10} 9}{\log_{10} 6}$

G $\frac{\log_{10} 3}{\log_{10} 2}$

38 Find the product of the real roots of the equation

$$\left(\log_{10} x^2\right)^2 + \log_{10} x = 3$$

A $10^{-\frac{3}{2}}$

B 10^{-1}

C $10^{-\frac{1}{2}}$

D $10^{-\frac{1}{4}}$

E $10^{\frac{3}{5}}$

F 10^{1}

39 Find the y-coordinate of the points on the curve $y = x^2$ that are closest to the point $\left(0, \frac{9}{2}\right)$

A 0

B $\frac{1}{4}$

C $\frac{4}{3}$

D 4

E $\frac{9}{2}$

40 Find the maximum value of the gradient of the curve with equation

$$y = 2 - 4x + 4x^{\frac{3}{2}} - x^2$$

where $x > 0$

A -4

B $-\frac{8}{9}$

C $\frac{1}{2}$

D 2

E 4

END OF TEST

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